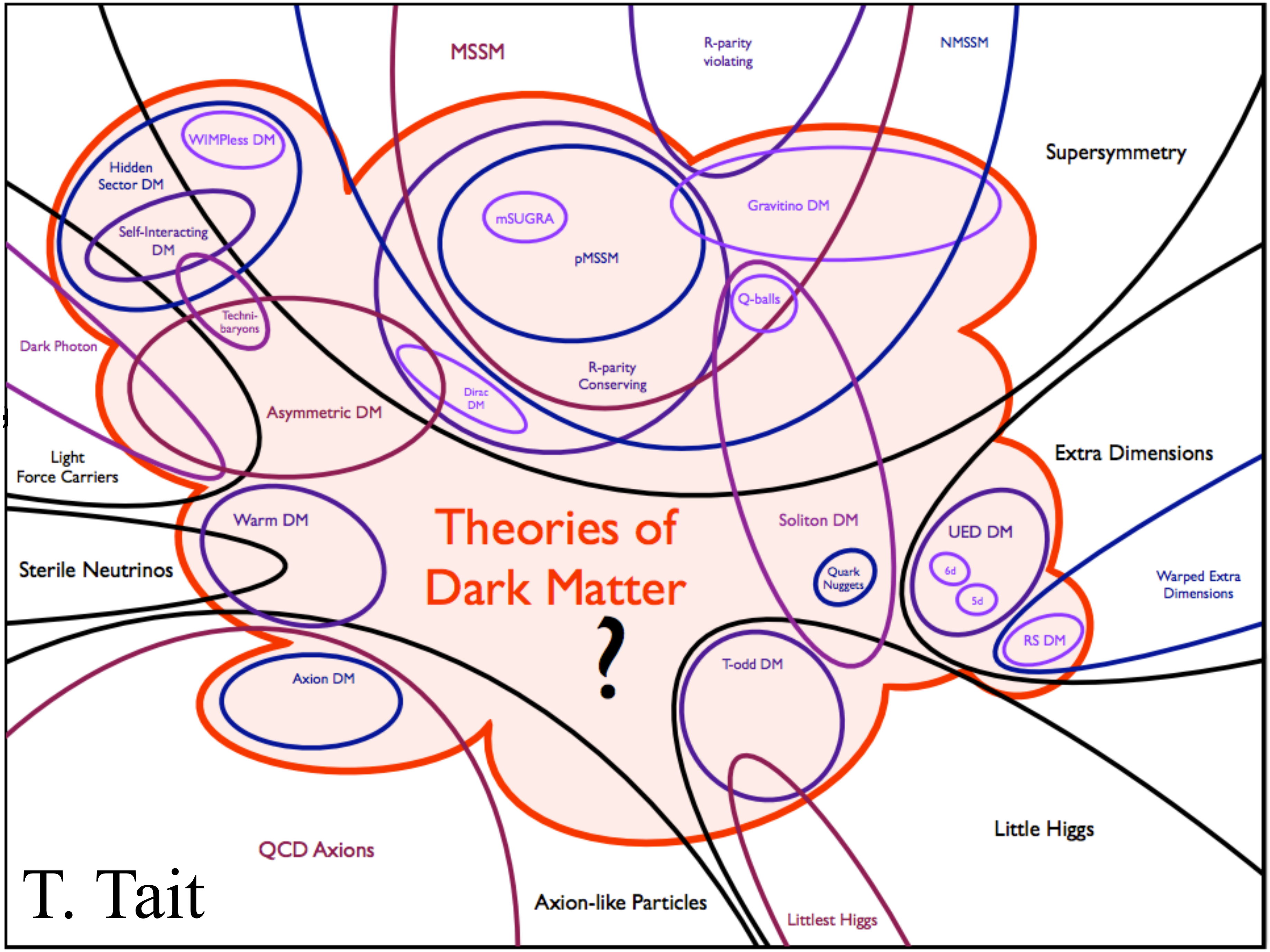
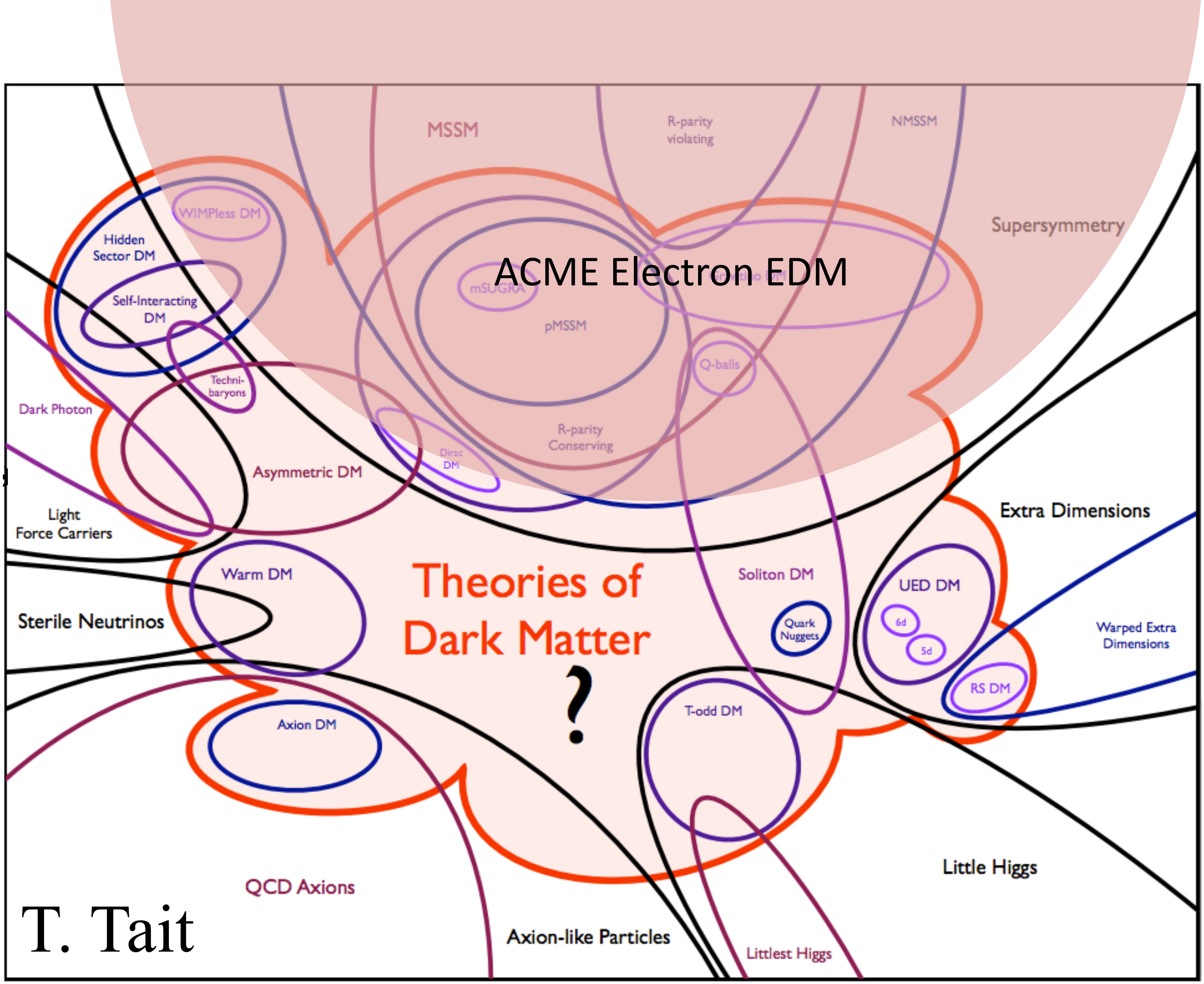
Axion Dark Matter Observables

in Variants of

the ACME Experiment

Brendon O'Leary, ACME Collaboration Meeting, March 12, 2015


MSSM
R-parity violating
NMSSM
Supersymmetry
WIMPless DM
Hidden Sector DM
Self-Interacting DM
Techni-baryons
mSUGRA
pMSSM
Gravitino DM
Q-balls
R-parity Conserving
Dirac DM
Dark Photon
Asymmetric DM
Light Force Carriers
Extra Dimensions
Warm DM
Theories of Dark Matter
?
Soliton DM
UED DM
6d
5d
Quark Nuggets
Sterile Neutrinos
Warped Extra Dimensions
RS DM
Axion DM
T-odd DM
Little Higgs
QCD Axions
T. Tait
Axion-like Particles
Littlest Higgs


ACME Electron EDM
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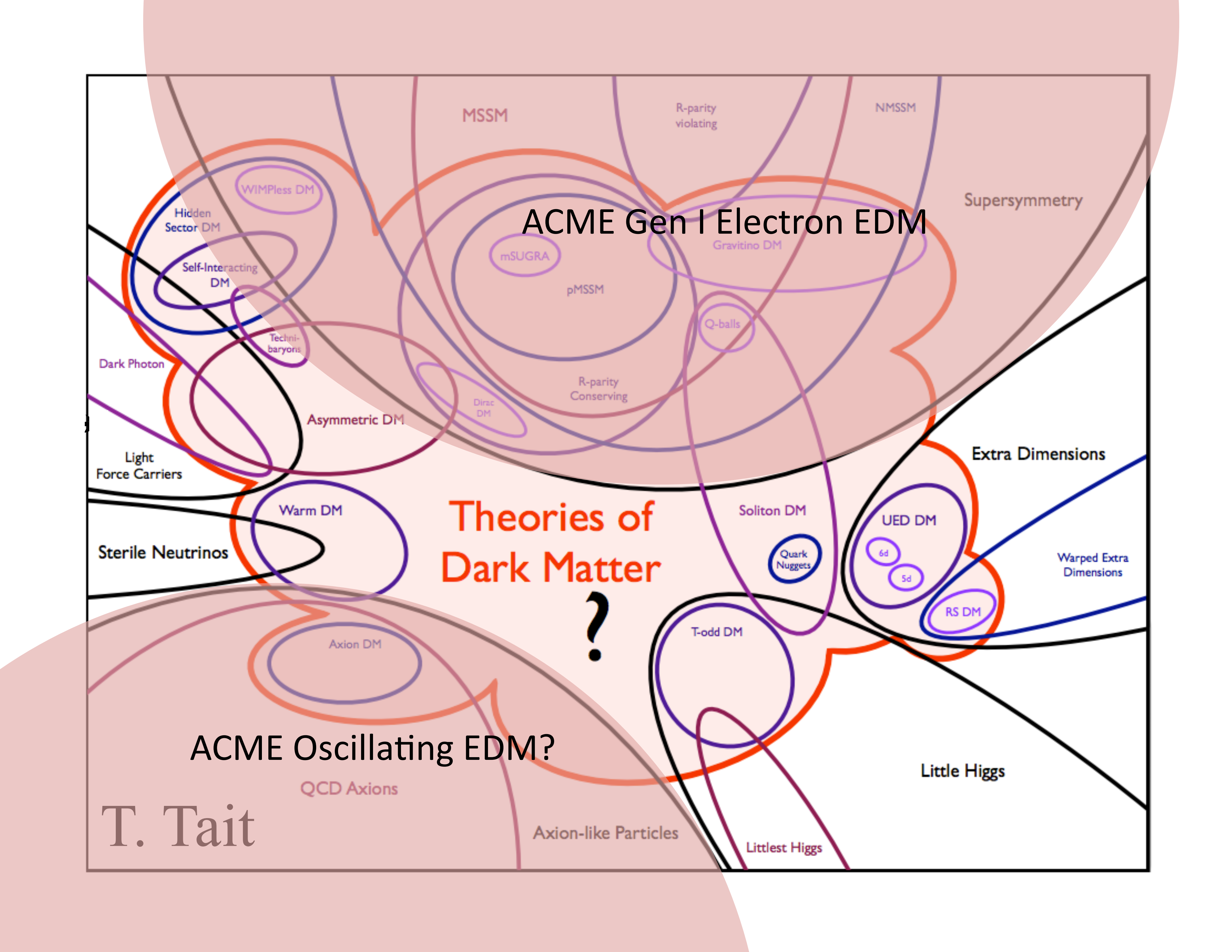

ACME Gen I Electron EDM
ACME Oscillating EDM?
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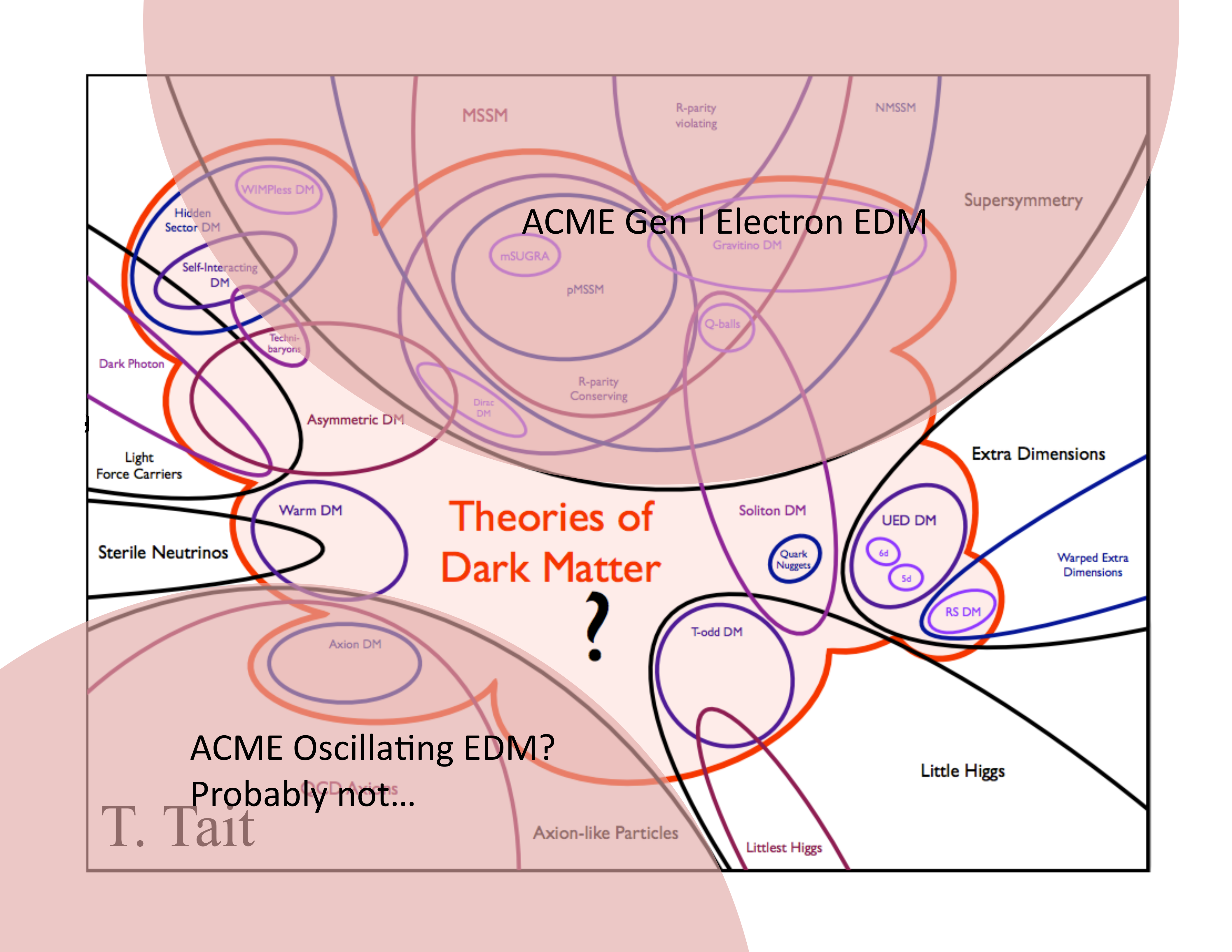
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Probably not...
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The Strong CP Problem

Most general gauge invariant lagrangian for strong interactions
(up to dimension 4 operators which ensures renormalizability)

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\;\mu\nu} + \bar{q}\left(\gamma_\mu D^\mu - \mathcal{M}_q\right) q + \frac{\alpha_s}{8\pi}\theta G^a_{\mu\nu}\tilde{G}^{a\;\mu\nu}$$

Usual gauge field energy

$\sim \mathcal{E}^2 + \mathcal{B}^2$

Usual fermion-gauge field interaction

Mass Term (or Mass Matrix)

Exotic Term that violates T and P symmetries

$\sim \mathcal{E} \cdot \mathcal{B}$

Measure of strong CP violation

$$\bar{\theta} = \theta + \mathrm{arg}\det\mathcal{M}_q < 10^{-10}$$

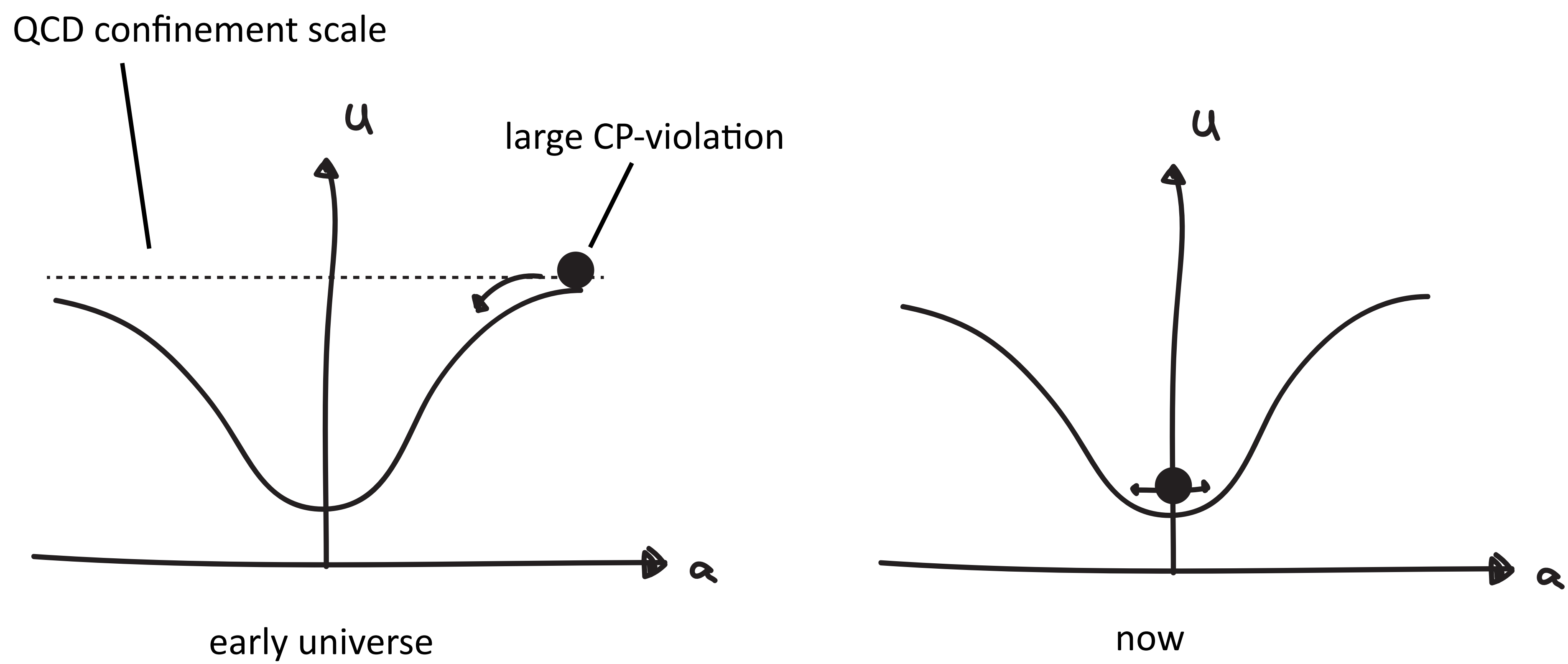
limit from Hg and neutron EDM experiments

Why is $\bar{\theta}$ so small?

Peccei-Quinn (PQ) Mechanism:

CP-violating parameter $\quad \bar{\theta} \rightarrow a$

- the axion is a pseudo-goldstone boson of a U(1) spontaneously broken symmetry at high energy scale f_a (decouples from regular matter early in the big bang)
- axion obtains a potential (and hence mass) in nonperturbative QCD.
- CP violation is large in early universe, but very small now.



Axions as Cold Dark Matter

Mostly defined entirely by two parameters: $m_a, \; f_a$

mass — symmetry breaking energy scale

- the parameters are related by mass generating mechanism

In most popular model mass is generated in nonperturbative QCD

$$m_a f_a \approx \Lambda_{QCD}^2 \approx (200 \text{ MeV})^2$$

- coherent classical scalar field

-amplitude from dark matter density
-decoherence from gravity-induced gradients

$$a = a_0 \cos\left(\omega t - \vec{k}\cdot\vec{x}\right)$$

$$\omega^2 = m^2 + k^2$$

$$\vec{k} = m\vec{v}$$

- amplitude and velocity estimated by astrophysics

$$\frac{1}{2} a_0^2 m_a^2 = \rho_{DM} \sim .3 \; \frac{\text{GeV}}{\text{cm}^3}$$

very specific prediction! 1 free parameter (m)

$$\bar{\theta} = \frac{a}{f_a} \sim 10^{-19} \cos\left(\vec{k}\cdot\vec{x} - \omega t\right)$$

Axion Interactions

QFT Lagrangians

(slight notation change here)

$$\mathcal{L}_{agg} = g_{agg} a \, G^{\mu\nu,\,a} \tilde{G}^{\;a}_{\mu\nu}$$

generates oscillating **nuclear electric dipole moments** and **magnetic quadrupole moments** in systems sensitive to θ_{QCD}

$$\mathcal{L}_{a\gamma\gamma} = g_{a\gamma\gamma} a \, \mathcal{E} \cdot \mathcal{B}$$

generates **axion-photon conversion** in the presence of a large magnetic field

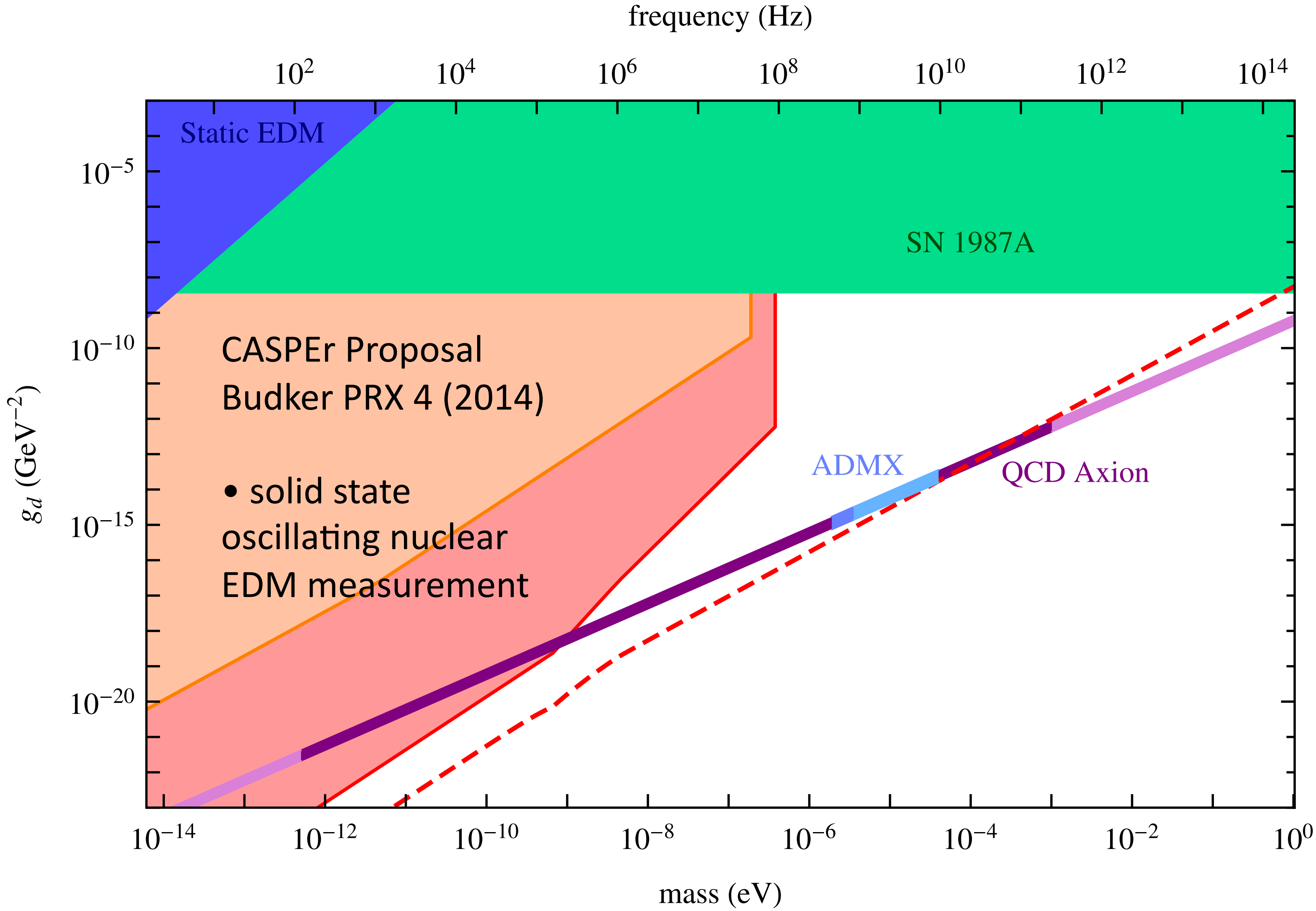
$$\mathcal{L}^0_{aee} = g^0_{agg} a \, \bar{e} m_e \gamma^5 e$$

$$\mathcal{L}^1_{aee} = g^1_{aee} (\partial_\mu a) \, \bar{e} \gamma^5 \gamma^\mu e$$

generate oscillating **atomic parity violation**, oscillating **lorentz violating interactions**, and oscillating **atomic electric dipole moments**

$$d_n = g_d a$$
$$g_d \approx \left[10^{-3}\mathrm{GeV}^{-1}\right] g_{agg}$$

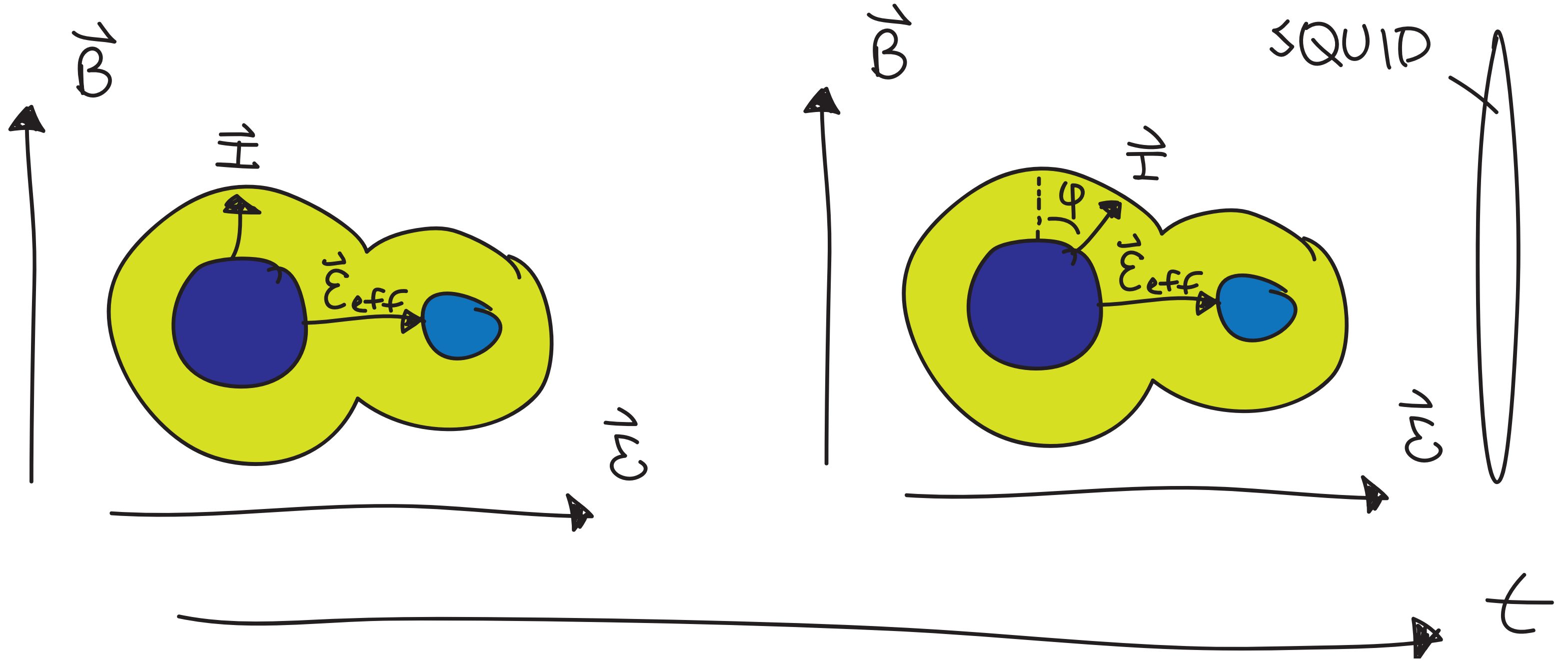
Gluon Coupling



search for oscillating nuclear moments in systems sensitive to θ_{QCD}

Solid State
Oscillating EDM Measurement

Budker D, Graham PW, Ledbetter M, Rajendran S, Sushkov AO. Proposal for a Cosmic Axion Spin Precession Experiment (CASPEr). Phys. Rev. X. 2014;4(2):021030



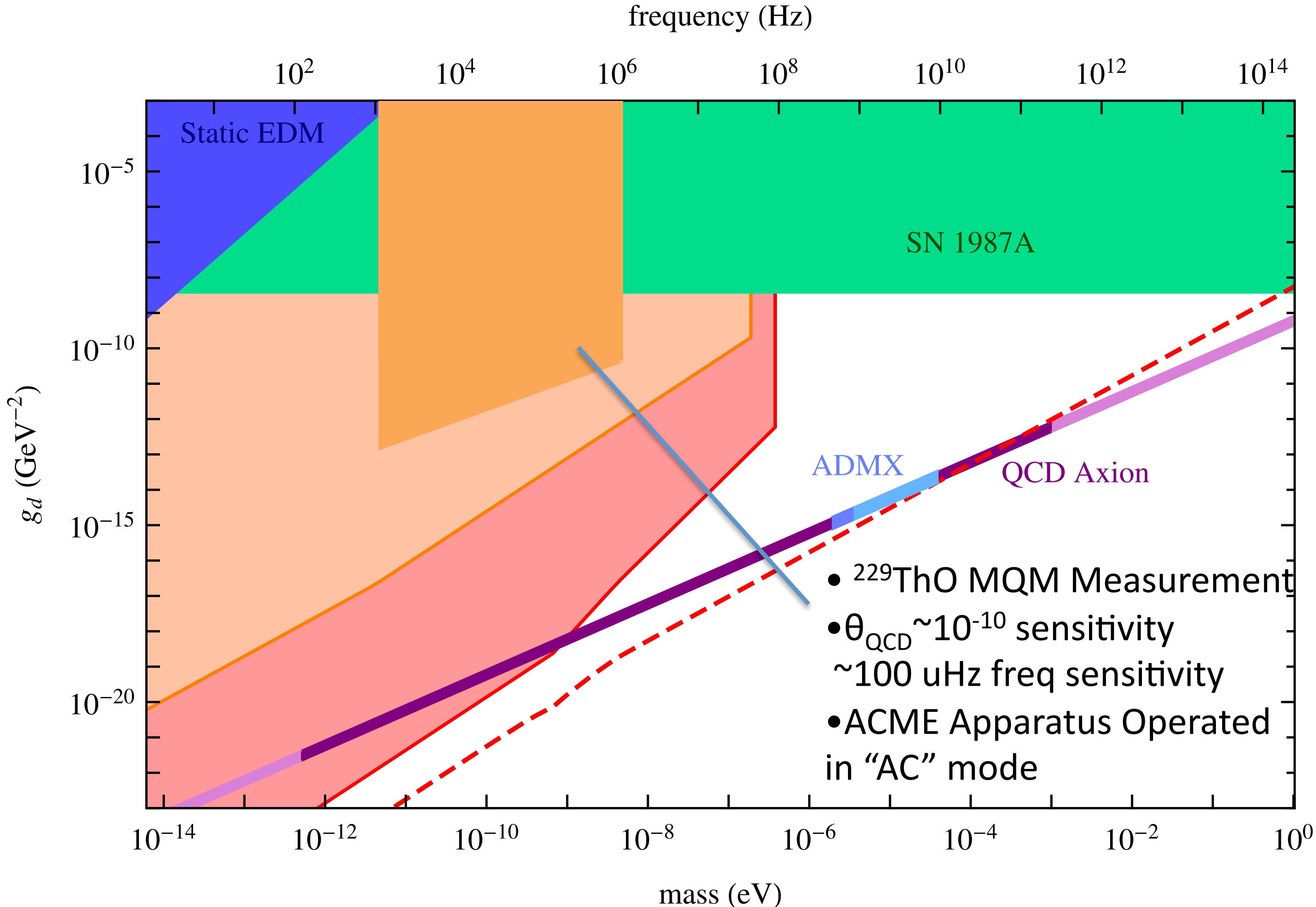
$$\text{magnetization}\,(t) \approx n\mu\ \left(d_n \mathcal{E}_{\text{eff}} t\right) \text{sinc}\left[\left(2\mu\mathcal{B} - m_a c^2\right) t\right] \sin\left(2\mu\mathcal{B}t\right)$$

B-induced magnetization

resonant behavior

temporal oscillation

Gluon Coupling

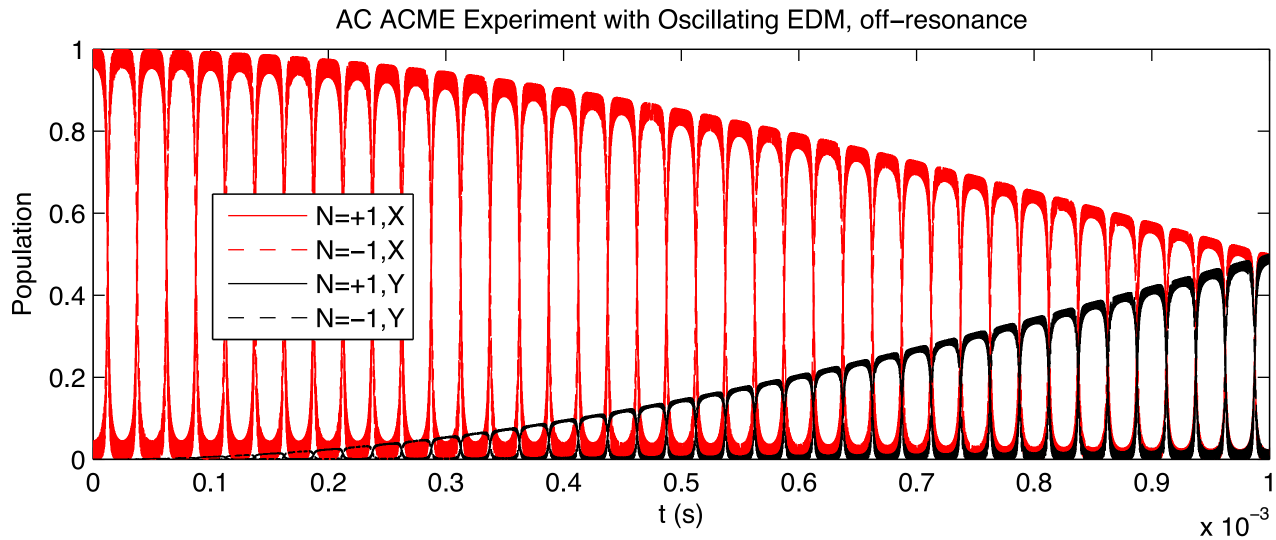


- ^{229}ThO MQM Measurement
- θ_{QCD}~10^{-10} sensitivity ~100 uHz freq sensitivity
- ACME Apparatus Operated in "AC" mode

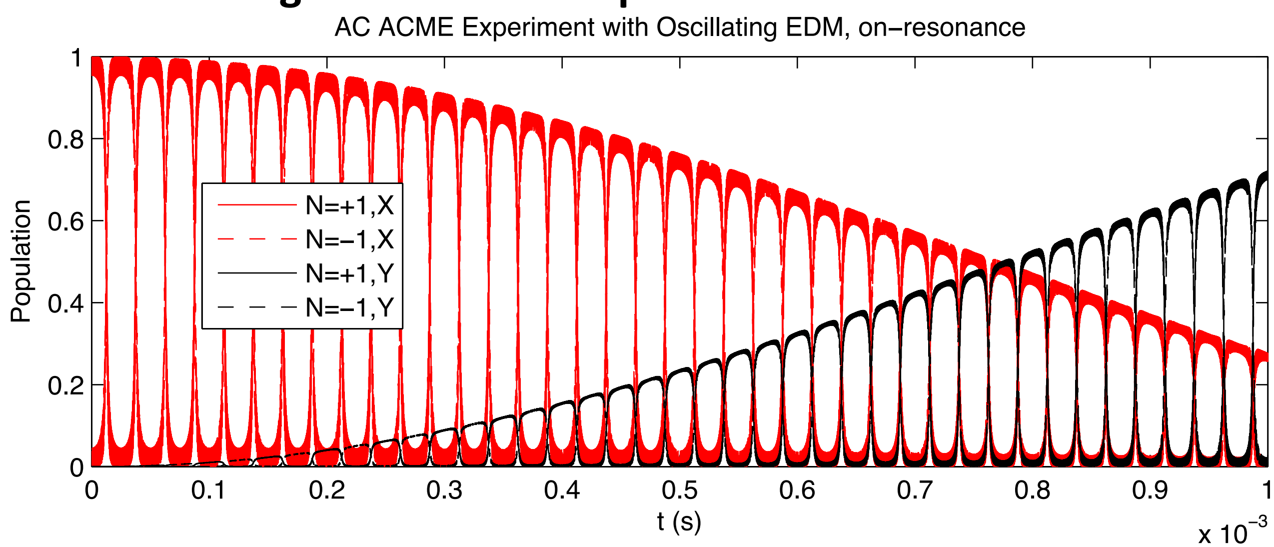
7 order of magnitude away from QCD axion

An AC ACME Experiment

- **Oscillate the Electric Field Adiabatically so that Molecules Stay in the Same Eigenstate**



- **If the frequency and phase of the electric field oscillation matches the axion field (within 1/t) then a DC phase shift is registered in the experiment**



An AC ACME Experiment

- **Resonant signal**

$$
\begin{aligned}
d_e\left(t\right) =& d_e^{(0)}\sin(\omega_d t) \\
\langle \tilde{\mathcal{N}}\tilde{\mathcal{E}}\left(t\right)\rangle \approx& \sin(\omega_{\mathcal{E}}t+\varphi)
\end{aligned}
$$

$$
\begin{aligned}
\phi\left(\varphi\right) \approx& \int_0^{\tau} d_e\mathcal{E}_{\mathrm{eff}}dt \\
\approx& \frac{1}{2}d_e^{(0)}\mathcal{E}_{\mathrm{eff}}\ \frac{\sin\left(\left(\omega_e-\omega_d\right)\tau+\varphi\right)-\sin\varphi}{\left(\omega_e-\omega_d\right)}
\end{aligned}
$$

- **Rejection of DC, and E-field phase independent measurement by fast phase switch**

$$
\begin{aligned}
\bar{\phi} =& \frac{1}{2}\sqrt{\left(\phi\left(\varphi\right)-\phi\left(\varphi+\pi\right)^2+\left(\phi\left(\varphi+\pi/2\right)-\phi\left(\varphi+3\pi/2\right)^2}} \\
\approx& \frac{1}{2}d_e\mathcal{E}_0\tau\left|\mathrm{sinc}\left(\frac{1}{2}\left(\omega_e-\omega_d\right)\tau\right)\right|
\end{aligned}
$$

- **Possible systematic errors?**
  - g factor difference between omega doublets coupling to AC magnetic field noise

$$
\bar{\phi}^{\mathrm{syst}}\left(\Delta g\right) = \frac{1}{2}g^{\mathcal{N}|\mathcal{E}|}\mu_{\mathcal{B}}\mathcal{B}^{\mathrm{nr}}(2\omega_{\mathrm{e}})\tau
$$

  - what are the effects of residual stray EM fields? Non-uniformity of AC field? …

• **Technical considerations:**

- Time response of field plates needs to be carefully characterized…

- Gen II field plates might be faster than Gen I plates (resistivity/square is smaller?!)

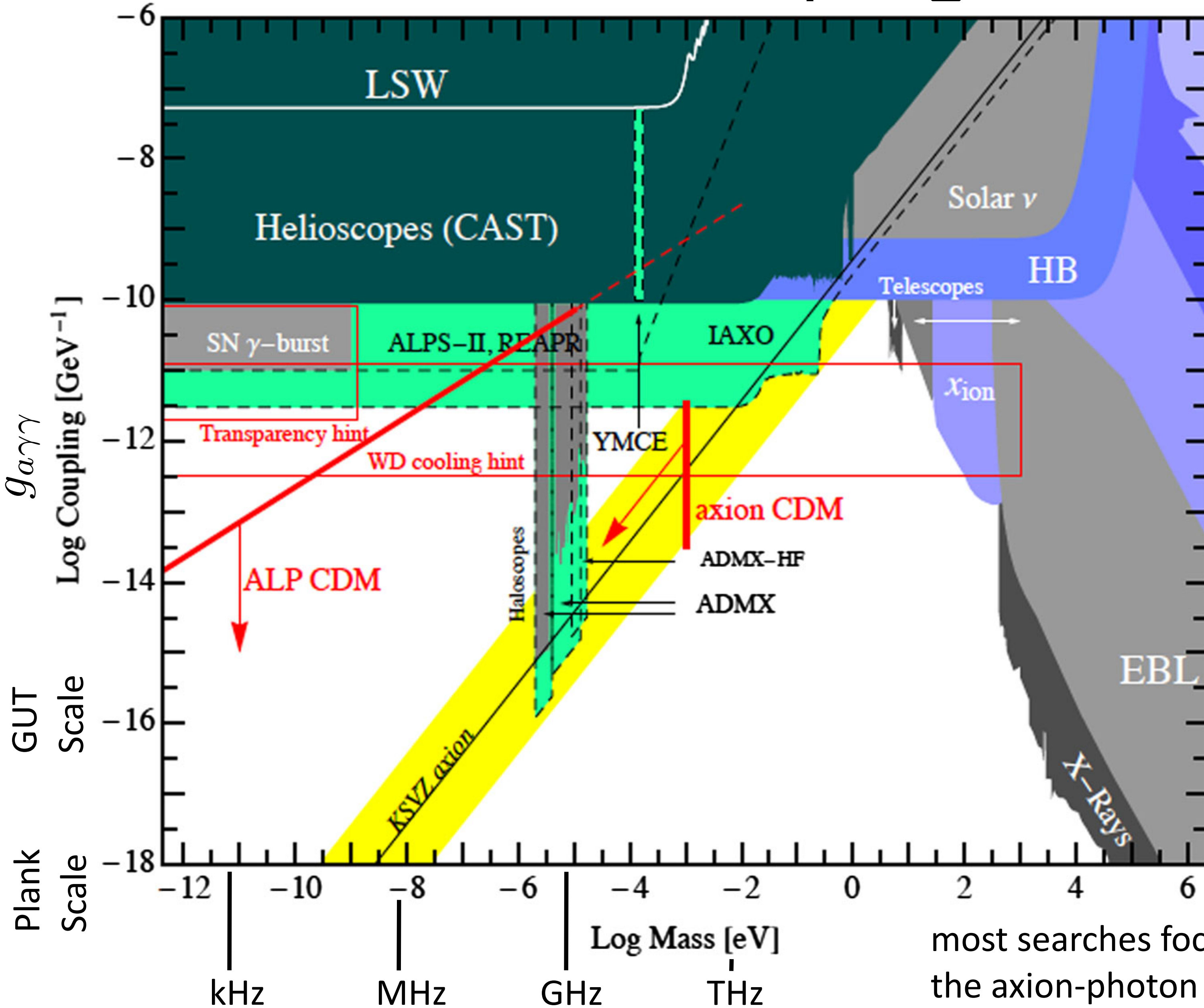
- Might be able to replace E-field supply with AC supply or capacitively couple in the AC (need about 2V amplitude at 1kHz-1MHz)

-Higher J is better (larger omega doublet splitting implies larger possible frequency band, but will require larger electric field amplitude to reach saturation)

-oscillating electric field generates oscillating transverse magnetic field via maxwells equations (could this do bad things?)

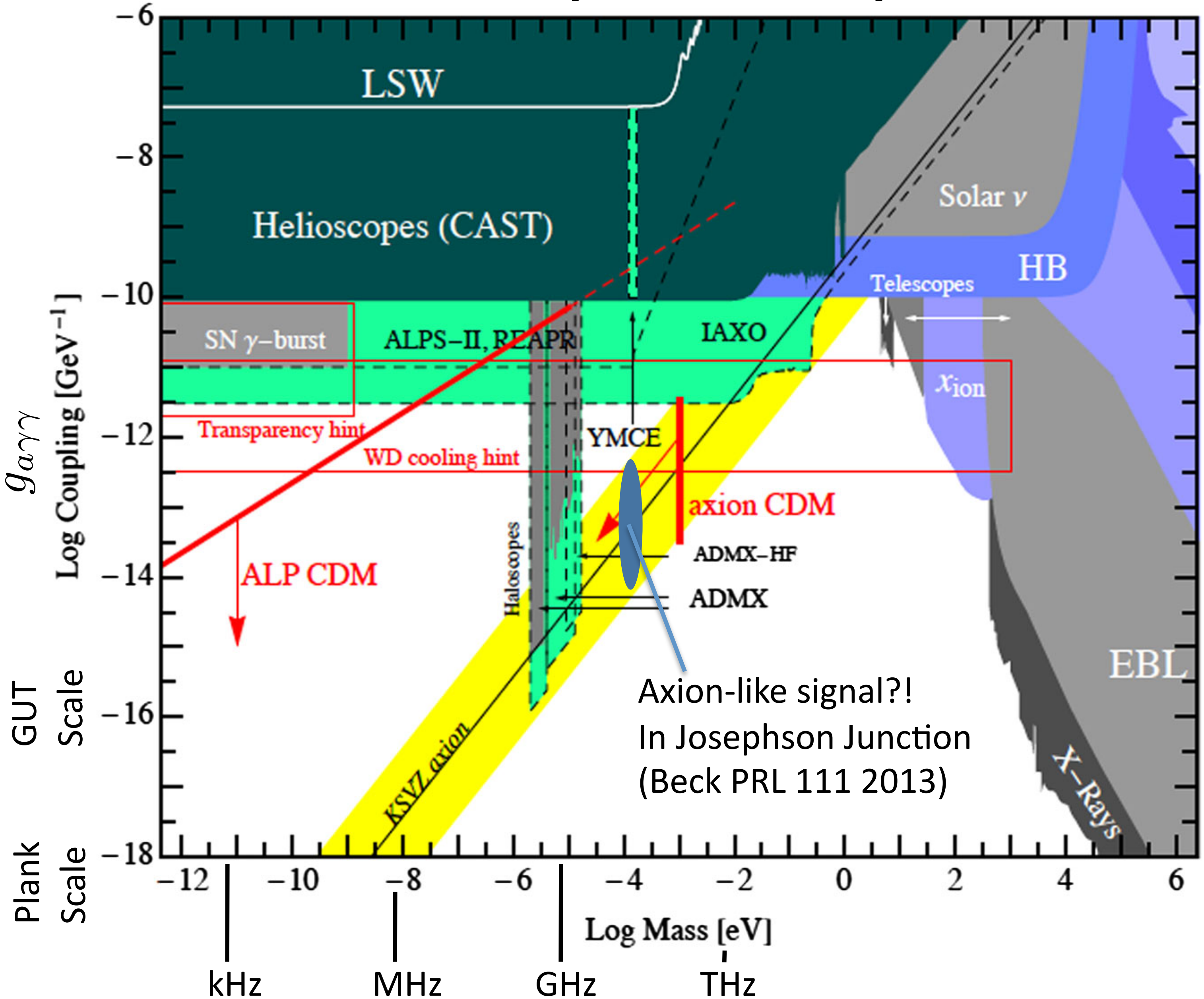
…

Photon Coupling

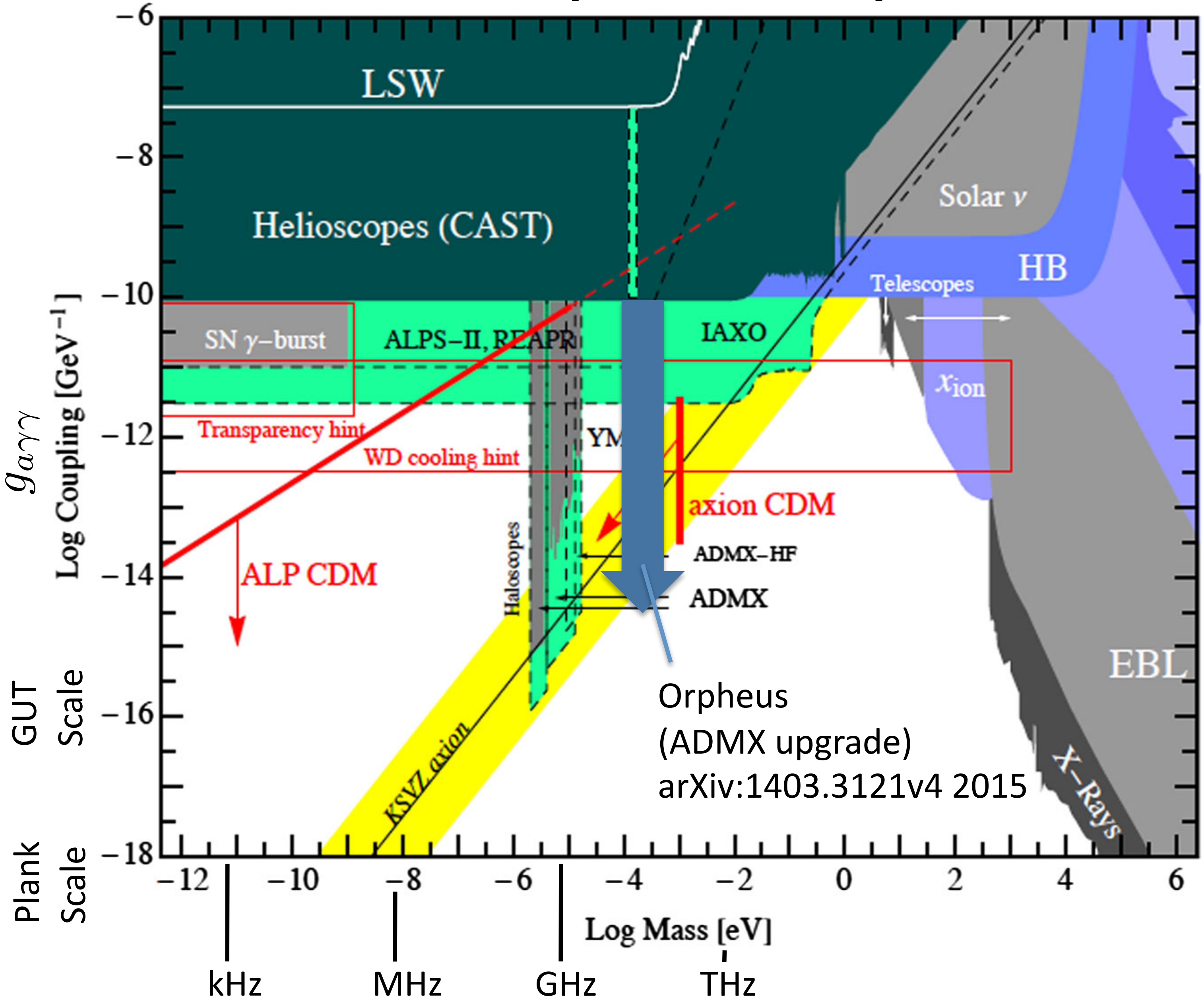


most searches focus on the axion-photon coupling

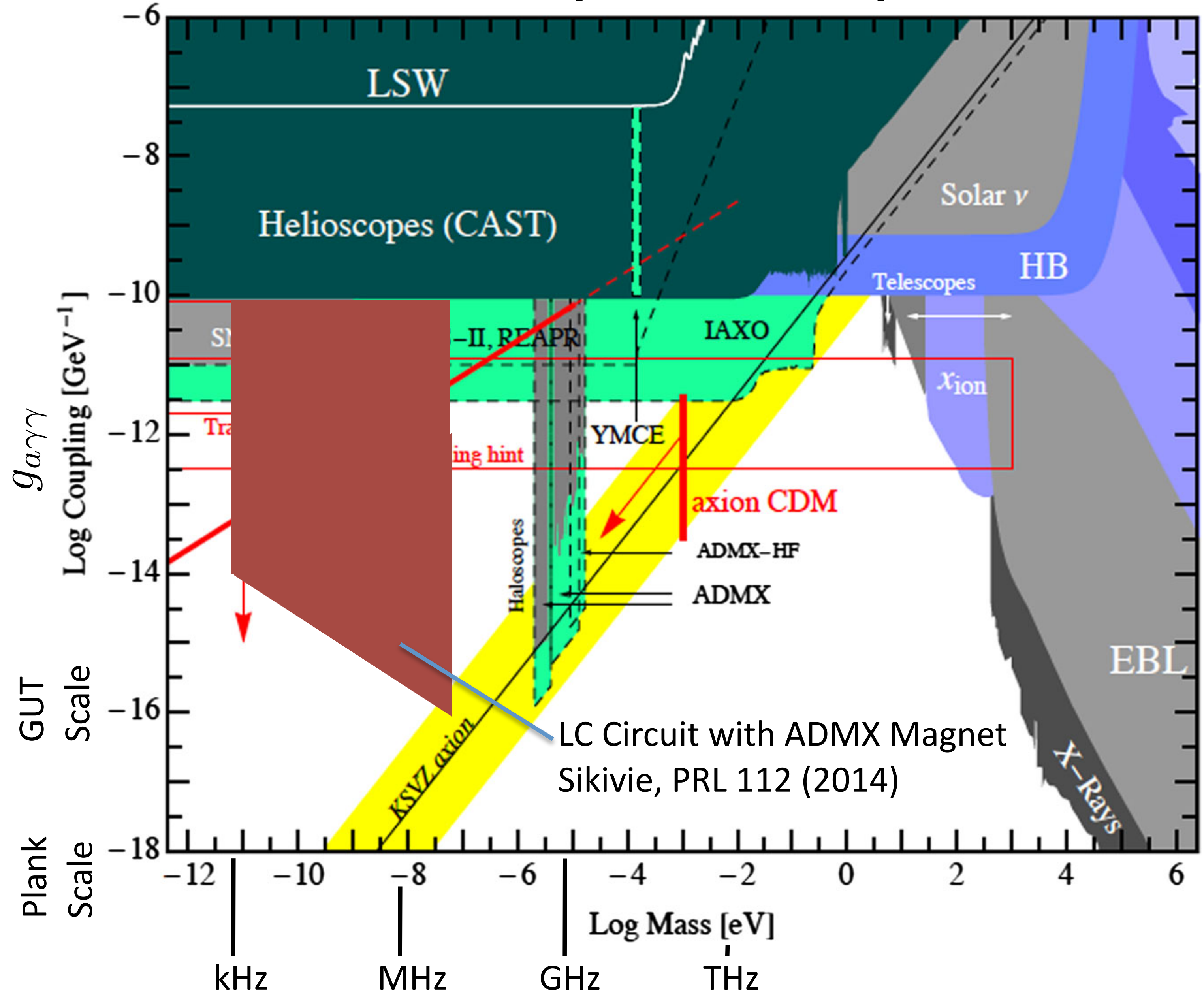
Past and Proposed Experiments



Past and Proposed Experiments



Past and Proposed Experiments



Axion Electrodynamics

$$\mathcal{L} = g_{a\gamma\gamma} a \mathcal{E} \cdot \mathcal{B}$$
$$\theta \equiv g_{a\gamma\gamma} a$$

Pure 4-Divergence for constant θ
(all physical effects proportional to axion derivatives)

$$\nabla \cdot \mathcal{E} = \rho - \nabla\theta \cdot \mathcal{B}$$
$$\nabla \times \mathcal{E} = -\dot{\mathcal{B}}$$
$$\nabla \cdot \mathcal{B} = 0$$
$$\nabla \times \mathcal{B} = \dot{\mathcal{E}} + J + \dot{\theta}\mathcal{B} + \nabla\theta \times \mathcal{E}$$
$$\nabla^2 a - \ddot{a} = -g_{a\gamma\gamma}\mathcal{E} \cdot \mathcal{B}$$

Maxwell's Equations

axion gradient-B field source
(ignore- effects are smaller than temporal-derivative coupling by v/c~10^{-4})

axion temporal derivative source
(dominant source assumed in most axion dark matter proposals)

axion gradient-E field source
(ignore unless E>10^4 B)

photon-axion conversion
(ignore- effects on electromagnetic fields are quadratic in a small coupling)

Driven Wave Equations

$$\nabla^2 \mathcal{E}_1 - \ddot{\mathcal{E}}_1 = \ddot{\theta}\mathcal{B}_0$$
$$\nabla^2 \mathcal{B}_1 - \ddot{\mathcal{B}}_1 = -\dot{\theta}\left(\nabla \times \mathcal{B}_0\right)$$

perturbative solution
0 – independent of axion field
1 – proportional to axion field

Separate Time Oscillatory Part and Solve By Helmholtz Greens Function

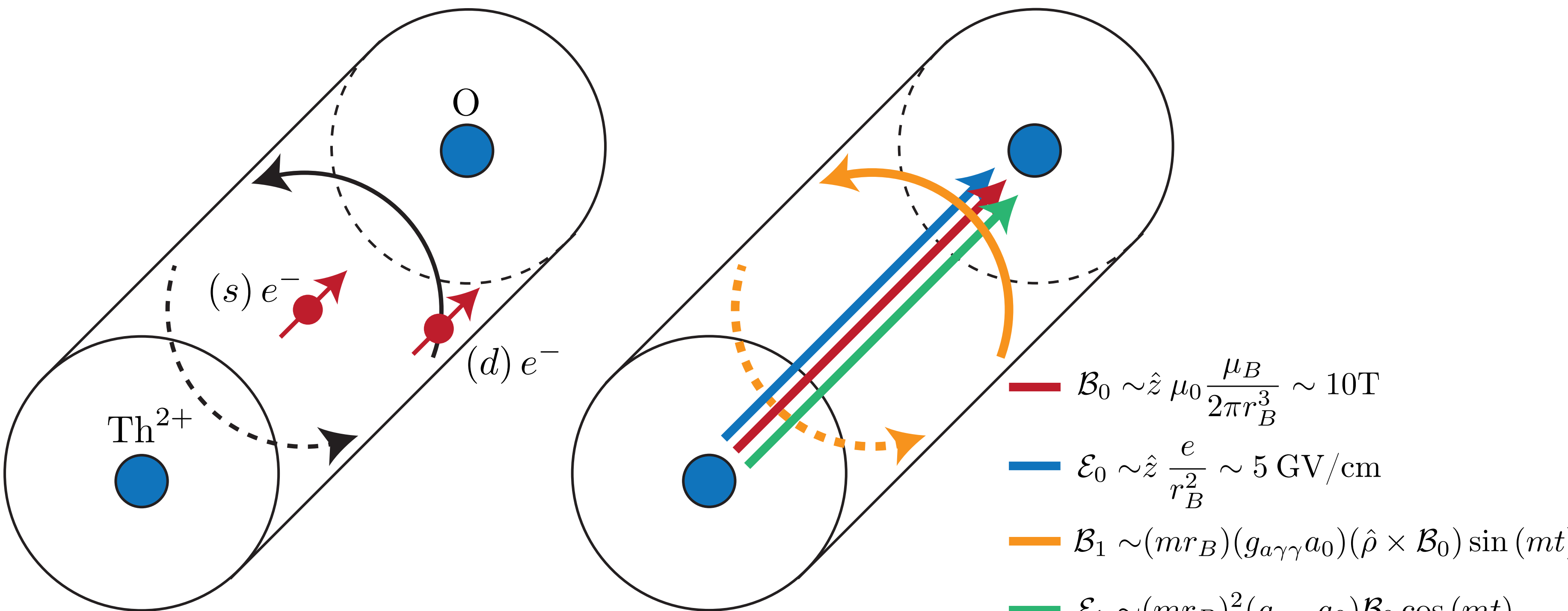
$$\theta = g_{a\gamma\gamma} a_0 \cos(mt)$$

$$\mathcal{E}_1(r) = m^2 g_{a\gamma\gamma} a_0 \cos(mt) \int d^3r' \frac{e^{-im|r-r'|}}{4\pi |r-r'|} \mathcal{B}_0(r')$$

$$\mathcal{B}_1(r) = m g_{a\gamma\gamma} a_0 \sin(mt) \int d^3r' \frac{e^{-im|r-r'|}}{4\pi |r-r'|} \nabla \times \mathcal{B}_0(r')$$

Axion Electrodynamics Within ThO

looking for axion-photon interaction within the molecular frame



- Axion Induced Magnetic field modulation is perpendicular to axis, changes Ω
  -not sure how to measure this effect…

- Axion Induced Electric field modulation changes the molecular electric dipole moment
  -shows up as an oscillating ThO EDM…

$$\delta g_{a\gamma\gamma} \sim \frac{\delta E}{(m r_B)^2 a_0 D_H \mathcal{B}_0} \sim \left[5 \cdot 10^9 \mathrm{GeV}^{-1}\right] \left(\frac{\delta E}{10^{-4}\mathrm{Hz}}\right) \left(\frac{1\mathrm{MHz}}{m}\right)$$

25 orders of magnitude away from QCD axion!

Axion Electrodynamics Within ACME Experiment

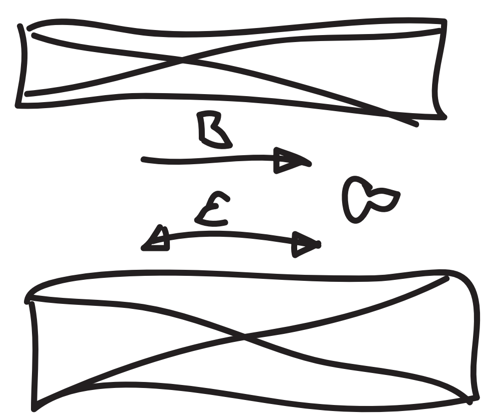
use the molecules to detect axion-photon coupling within the experiment

- If we had a 10 tesla magnet in the experiment, then the effect is much larger because the spatial extent of the field is much larger, and we can use as electric field sensors:

$$\delta g_{a\gamma\gamma} \sim \frac{\delta E}{(m r_B)^2 a_0 D_H \mathcal{B}_0} \sim \left[5 \cdot 10^{-11}\text{GeV}^{-1}\right] \left(\frac{\delta E}{10^{-4}\text{Hz}}\right) \left(\frac{1\text{MHz}}{m}\right)$$

- Now, only 5 orders of magnitude away from the QCD axion, but this is a completely different experiment, would explore some new parameter space, but not much.

- This is pretty much a standard axion search method, but here the molecules are the field detectors



Aside: Axion Electrodynamics With Magnetic Monopoles

Visinelli 2013 arXiv:1401.0709v1

- Maxwells equations take on a more symmetric form
- EM fields can be generated without derivative of axion field
- Axion field causes electric charge to take on oscillating magnetic monopole-like character

$$\begin{aligned}
&\nabla\cdot\left(\mathbf{E} - c\kappa\,\theta\,\mathbf{B}\right) = \rho_e/\epsilon_0,\\
&\nabla\times\left(c\mathbf{B} + \kappa\,\theta\,\mathbf{E}\right) = \partial_t\left(\mathbf{E} - c\kappa\,\theta\,\mathbf{B}\right)/c + c\mu_0\,\mathbf{J}_e,\\
&\nabla\cdot\left(c\mathbf{B} + \kappa\,\theta\,\mathbf{E}\right) = c\mu_0\,\rho_m,\\
&\nabla\times\left(\mathbf{E} - c\kappa\,\theta\,\mathbf{B}\right) = -\partial_t\left(c\mathbf{B} + \kappa\,\theta\,\mathbf{E}\right)/c - \mu_0\,\mathbf{J}_m,\\
&\Box\,\theta = -\frac{\kappa}{\mu_0\,c}\,\mathbf{E}\cdot\mathbf{B} - \frac{\partial U(\theta)}{\partial\theta}.
\end{aligned}$$

- In this exotic model, an AC ACME experiment is sensitive to the QCD axion (oscillating H state EDM caused by effective electric field generating an oscillating internal magnetic field, causing spin precession)

$$\delta g_{a\gamma\gamma} = \frac{\delta E}{a_0 g_S \mu_B \mathcal{E}_{\rm eff}} \approx \left[10^{-19}\mathrm{GeV}^{-1}\right]\left(\frac{\delta E}{10^{-4}\,\mathrm{Hz}}\right)\left(\frac{m}{\mathrm{kHz}}\right)$$

- Lots of people think magnetic monopoles exist, but better not to assume out-right

Axion-Electron Coupling

$$\mathcal{L}^0_{aee} = g^0_{agg} a\, \bar{e} m_e \gamma^5 e$$

$$\mathcal{L}^1_{aee} = g^1_{aee} (\partial_\mu a)\, \bar{e} \gamma^5 \gamma^\mu e$$

both of these couplings have roughly the same effect

$$H = g^{(1)}_{aee} \left\{ \dot{a}\, \frac{p_e}{m_e} \cdot \sigma_e + \nabla a \cdot \sigma_e \right\}$$

atomic parity violation
(from axion time derivative coupling)

axion wind effect
(oscillating spin precession around the cosmic axion momentum direction)

"PV Induced EDM"

- Oscillating parity violating term mixes opposite parity states.
- Apply small stark mixing to result in energy shift linear in electric field
- Axion frequency assumed to be close to omega doublet splitting, modulate electric field magnitude in partial polarization regime.
- Relativistic calculations are probably required to estimate sensitivity to this effect

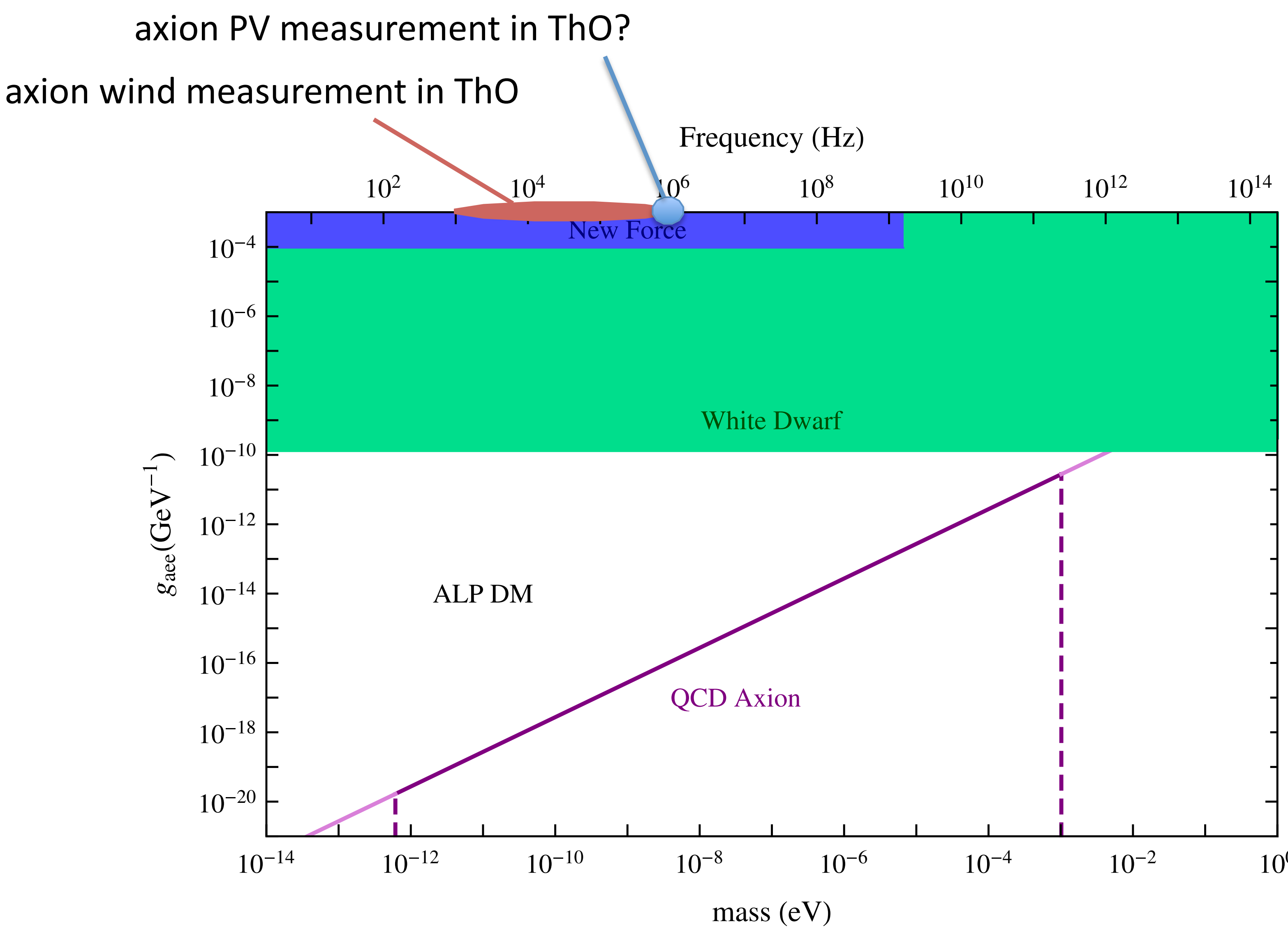
Axion Wind Spin Precession
perform an AC spin precession measurement

(modulate electric field to drive electrons into and out of polarization along the electric field axis)

$$E = M\, m_a\, (g a_0) \left(\frac{\vec{v} \cdot \hat{z}}{c} \right) \sin{(mt)}$$

Axion-Electron Coupling

Not very sensitive to axion-like particles



Conclusion

Model	Coupling Constant	Mass Scaling		
QCD Axion	$g \approx (10^{-19}\ \mathrm{GeV}^{-1})$	$\times\left(\frac{m}{1\ \mathrm{kHz}}\right)$		(favored model)
ALP Models	$g < (10^{-10}\ \mathrm{GeV}^{-1})$	$\begin{pmatrix} m < \mathrm{THz} \\ m > \mathrm{kHz} \end{pmatrix}$		(to say something interesting)
Experiment Details	**Coupling Sensitivity**	**Mass Scaling**	**Sensitivity**	
^{229}ThO AC MQM	$\delta g_{agg} \sim \left[3 \times 10^{-10}\ \mathrm{GeV}^{-1}\right]$	$\times\left(\frac{m}{1\ \mathrm{kHz}}\right)$	$\times\left(\frac{\delta E}{10^{-4}\mathrm{Hz}}\right)$	ALP level, but requires ^{229}ThO (expensive, radioactive)
$a\gamma\gamma$ inside ThO	$\delta g_{a\gamma\gamma} \sim \left[5 \times 10^{+9}\ \mathrm{GeV}^{-1}\right]$	$\times\left(\frac{1\ \mathrm{MHz}}{m}\right)$	$\times\left(\frac{\delta E}{10^{-4}\mathrm{Hz}}\right)$	Horrible axion wavelength/molecule size mismatch
$a\gamma\gamma$ inside ThO with MM	$\delta g_{a\gamma\gamma} \sim \left[1 \times 10^{-19}\ \mathrm{GeV}^{-1}\right]$	$\times\left(\frac{m}{1\ \mathrm{kHz}}\right)$	$\times\left(\frac{\delta E}{10^{-4}\mathrm{Hz}}\right)$	QCD axion level!!, but requires magnetic monopoles :(
$a\gamma\gamma$ from huge magnet	$\delta g_{a\gamma\gamma} \sim \left[5 \times 10^{-11}\ \mathrm{GeV}^{-1}\right]$	$\times\left(\frac{1\ \mathrm{MHz}}{m}\right)$	$\times\left(\frac{\delta E}{10^{-4}\mathrm{Hz}}\right)$	ALP level, requires huge magnetic (experiment change)
aee in ThO (PV induced "EDM")	$\delta g_{aee} \sim \left[5 \times 10^{-3}\mathrm{GeV}^{-1}\right]$	$\times\left(\frac{1\ \mathrm{MHz}}{m \sim \Delta \sim D\mathcal{E}}\right)^3$	$\times\left(\frac{\delta E}{10^{-4}\ \mathrm{Hz}}\right)$	Not Very Interesting, Need Relativistic Calc.?
aee axion-wind in ThO	$\delta g_{aee} \sim \left[2 \times 10^{-3}\ \mathrm{GeV}^{-1}\right]$	$\times 1$	$\times\left(\frac{\delta E}{10^{-4}\ \mathrm{Hz}}\right)$	12-24 Hour and Yearly Modulation of Signal

most promising:

- ^{229}ThO AC MQM Experiment
- AC Stark Shift Modulation in Large Magnetic Field

} **Both require significant changes to existing experiment**

Probably wouldn't probe any interesting parameter space with an "AC Electron EDM" search in ThO with existing apparatus

In favor of performing an AC ACME Experiment:

• 'Born doubted that the deflection experiment would prove worthwhile. Gerlach's response was to quote a favorite saying "No experiment is so dumb, that it should not be tried" '

• We don't have much of an idea of what dark matter might be yet – if this observable might be related to dark matter, and if it is easy to check, then it might we might as well.

• Theories don't predict that we would see anything. But theories might change – in the future we might regret not having performed this measurement.

Lorentz Violating Observables

(Standard Model Extension)

Photon Sector

$$\mathcal{L} = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\left(k_{AF}\right)^{\kappa}\epsilon_{\kappa\lambda\mu\nu}A^{\lambda}F^{\mu\nu} - \frac{1}{4}\left(k_F\right)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu}$$

relativistic lagrangian

P,T violating, might show up in EDM signal

Kostelecky (2009) arXiv: 0905.0031v2

$$\mathcal{D} = (1 + \kappa_{\mathcal{D}\mathcal{E}})\mathcal{E} + \boxed{\kappa_{DB}\mathcal{B}} + 2\vec{k}_{AF} \times \mathcal{A}$$
$$\mathcal{H} = (1 + \kappa_{\mathcal{H}\mathcal{B}})\,\mathcal{B} + \boxed{\kappa_{\mathcal{H}\mathcal{E}}\mathcal{E}} + 2\vec{k}_{AF}\mathcal{A}_0$$

modified field definitions (κ are matrices)

$$\nabla \cdot \mathcal{D} = 0$$
$$\nabla \cdot \mathcal{B} = 0$$
$$\nabla \times \mathcal{H} - \partial_0 \mathcal{D} = 0$$
$$\nabla \times \mathcal{E} - \partial_0 \mathcal{B} = 0$$

modified maxwells equations

Fermion Sector

Kostelecky (1999) arXiv:hep-ph/9908504v1

$$\mathcal{L} = \frac{1}{2}i\bar{\psi}\Gamma_{\nu}\overleftrightarrow{\partial^{\nu}}\psi - \bar{\psi}M\psi$$
$$M = m + a_{\mu}\gamma^{\mu} + b_{\mu}\gamma^{5}\gamma^{\mu} + \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu}$$
$$\Gamma_{\nu} = \gamma_{\nu} + c_{\mu\nu}\gamma^{\mu} + d_{\mu\nu}\gamma^{5}\gamma^{\mu} + e_{\nu} + if_{\nu}\gamma^{5} + \frac{1}{2}g_{\lambda\mu\nu}\sigma^{\lambda\mu}$$

relativistic lagrangian

$$H = A + B_i\sigma_i + C_i p_i + D_{ij}p_i\sigma_j + E_{ij}p_i p_j + F_{ijk}p_i p_j \sigma_k$$

non-relativistic hamiltonian

"EDM Sector"

"cosmic EDM field"

$$H = b_i\sigma_i + \boxed{d_{ij}\sigma_i\mathcal{E}_j} + \mu_{ij}\sigma_i\mathcal{B}_j$$

non-relativistic hamiltonian
"in the spirit of the SME"

Altarev et al. EPL 92 (2010)

Table S3. Maximal sensitivities for the photon sector

$d=3$	**Coefficient**	**Sensitivity**		
	$k^{(3)}_{(V)00}$	10^{-43} GeV		
	$k^{(3)}_{(V)10}$	10^{-42} GeV		
	$\mathrm{Re}\, k^{(3)}_{(V)11}$	10^{-42} GeV		
	$\mathrm{Im}\, k^{(3)}_{(V)11}$	10^{-42} GeV		
$d=4$	**Coefficient**	**Sensitivity**	**Coefficient**	**Sensitivity**
	$(\tilde{\kappa}_{e+})^{XY}$	10^{-32}	$(\tilde{\kappa}_{e-})^{XY}$	10^{-17}
	$(\tilde{\kappa}_{e+})^{XZ}$	10^{-32}	$(\tilde{\kappa}_{e-})^{XZ}$	10^{-17}
	$(\tilde{\kappa}_{e+})^{YZ}$	10^{-32}	$(\tilde{\kappa}_{e-})^{YZ}$	10^{-17}
	$(\tilde{\kappa}_{e+})^{XX} - (\tilde{\kappa}_{e+})^{YY}$	10^{-32}	$(\tilde{\kappa}_{e-})^{XX} - (\tilde{\kappa}_{e-})^{YY}$	10^{-17}
	$(\tilde{\kappa}_{e+})^{ZZ}$	10^{-32}	$(\tilde{\kappa}_{e-})^{ZZ}$	10^{-16}
	$(\tilde{\kappa}_{o-})^{XY}$	10^{-32}	$(\tilde{\kappa}_{o+})^{XY}$	10^{-13}
	$(\tilde{\kappa}_{o-})^{XZ}$	10^{-32}	$(\tilde{\kappa}_{o+})^{XZ}$	10^{-14}
	$(\tilde{\kappa}_{o-})^{YZ}$	10^{-32}	$(\tilde{\kappa}_{o+})^{YZ}$	10^{-14}
	$(\tilde{\kappa}_{o-})^{XX} - (\tilde{\kappa}_{o-})^{YY}$	10^{-32}		
	$(\tilde{\kappa}_{o-})^{ZZ}$	10^{-32}	$\tilde{\kappa}_{\mathrm{tr}}$	10^{-14}
Isotropic	**Coefficient**	**Sensitivity**		
	$k^{(3)}_{(V)00}$	10^{-43} GeV		
	$c^{(4)}_{(I)00} = \sqrt{4\pi}\tilde{\kappa}_{\mathrm{tr}}$	10^{-14}		
	$k^{(5)}_{(V)00}$	10^{-34} GeV^{-1}		
	$c^{(6)}_{(I)00}$	10^{-21} GeV^{-2}		
	$k^{(7)}_{(V)00}$	10^{-27} GeV^{-3}		
	$c^{(8)}_{(I)00}$	10^{-24} GeV^{-4}		
	$k^{(9)}_{(V)00}$	10^{-21} GeV^{-5}		

From Kostelecky's amazing list of limits on Lorentz Invariance Violation!

If we are sensitive to this parameter, we could probably set a limit like:

$$\kappa^{(4)}_{o\pm} < d_e/\mu_B \sim 5 \times 10^{-18}$$

(Would require careful reanalysis of Gen I data looking for a particular Lorentz invariance signature)

If we are sensitive to this parameter, we could probably set a limit like:

$$\kappa^{(6)}_{o\pm} < d_e/\mu_B a_0^2 \sim 5 \times 10^{-29}\mathrm{GeV}^2$$