To whom it may concern,

An error was discovered in the data analysis in work of Johnson, Brahms and Newman theses. To be specific, an incorrect formula for the optical cross section was used, leading to errors in atom number in the work with iron, nickel, copper, cobalt, silver, dysprosium and holmium, and the collisional reorientation rate constant in the work of dysprosium.

Detailed derivations of the optical cross section can be founded in Ref. [1, 2, 4, 5].

$$\sigma_{\text{correct}} = \frac{3\lambda_0^2}{2\pi} \frac{\Gamma^2/4}{(\omega - \omega_0)^2 + \Gamma^2/4} |\langle J, M_J, 1, q | J', M_J' \rangle|^2 \tag{1}$$

$$\mathcal{F} = \frac{\sigma_{\text{correct}}}{\sigma_{\text{theses}}} \tag{2}$$

Element	$\mathcal{F}$
Ag (328 nm)	3
Co (241 nm)	9
Cu (325 nm)	3
Dy (405 nm)	$195/17 \approx 11.5$
Fe (248 nm)	7
Ho (411 nm)	15
Ni (232 nm)	9

For dysprosium, the updated two-body loss rate constant (g_R) is $(2.1 \pm 0.2) \times 10^{-11} \text{cm}^3\text{s}^{-1}$. Due to trap dynamics, the collisional reorientation rate constant could be a few times higher [3]. We claim that there is strong evidence for spin relaxation driven by electrostatic anisotropy in collisions between dysprosium atoms.

We also would like to point out an unfortunate typographical error in the abstract of Newman's thesis. Dysprosium was adiabatically, *not evaporatively*, cooled to 50 mK.

At the time of writing, the dysprosium results have yet to be published. In the meantime, please cite the dysprosium results as "B. Newman, N. Brahms, Y. S. Au, C. Johnson, C. B. Connolly, J. M. Doyle, D. Kleppner, and T. Greytak, (in preparation)."

For holmium, the updated result of the collisional reorientation rate constant (g_R) is $(1.1 \pm 0.2) \times 10^{-11}\text{cm}^3\text{s}^{-1}$. We consider the result to be preliminary and do not draw any conclusions about the spin relaxation mechanism.

Regards,

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References

[1] Dmitry Budker, Derek Kimball, and David DeMille. *Atomic physics: An exploration through problems and solutions.* Oxford University Press, USA, 2 edition, November 2008.

[2] Claude Cohen-Tannoudji, Jacques Dupont-Roc, and Gilbert Grynberg. *Atom-Photon Interactions: Basic Processes and Applications.* Wiley-VCH, March 1998.

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Introduction

In order to extract information such as atom number, temperature and state distribution from the measured trapped spectrum, a knowledge of the optical absorption cross section is needed. The cross section can be measured or calculated. Detail derivations can be founded in Ref. [1, 2, 3, 4]. Here we provide a summary of the results that are relevant to our data analysis. We will focus on the case of a two-level atom.

Two-state atom

Consider a laser beam traveling through a medium along the z-axis.

$$\frac{dI}{dz} = -(N_g - N_e)\sigma(\omega)I(\omega) \tag{1}$$

The right-hand side of Eq. 1 is the rate of simulated absorption in excess of simulated emission, which in steady state equals to the rate of spontaneous emission.

$$-(N_g - N_e)\sigma I = N_e \Gamma \hbar \omega$$

Hence,

$$\sigma(\omega) = \frac{N_e}{N_e - N_g}\frac{\Gamma \hbar \omega}{I} \tag{2}$$

The atomic state populations are given by the steady state solution to the optical Bloch equation [3].

$$\rho_{ee} = \frac{N_e}{N} \tag{3}$$

$$w = \frac{N_e - N_g}{N} \tag{4}$$

We have,

$$\sigma(\omega) = \frac{\Omega^2/4}{(\omega - \omega_0)^2 + \Gamma^2/4}\frac{\Gamma \hbar \omega}{I} \tag{5}$$

$$= L(\omega)\Gamma\frac{\pi}{2}\frac{\hbar \omega}{I}\frac{\Omega^2}{\Gamma} \tag{6}$$

where $L(\omega)$ is the Lorentzian lineshape.

$$I = \frac{n\hbar\omega c}{V} \tag{7}$$

$$\frac{\hbar\omega}{I} = \frac{V}{nc} \tag{8}$$

The Rabi frequency is defined to be

$$\hbar\Omega = -\vec{d} \cdot \vec{E} \tag{9}$$

Using $\epsilon_0 E^2 = n\hbar\omega/V$,

$$\Omega^2 = \frac{n\omega_0}{\epsilon_0 \hbar V}|\langle g|\vec{d} \cdot \hat{u}|e\rangle|^2 \tag{10}$$

where $\hat{u}$ is the spherical unit vector.

The spontaneous emission rate Γ can be calculated using Fermi's Golden rule.

$$\Gamma = \frac{2\pi}{\hbar} |\langle i|H'|f\rangle|^2 \rho_f(\hbar\omega)\delta(\hbar\omega_0) \tag{11}$$

where $|i\rangle = |e\rangle|n-1\rangle$, $|f\rangle = |g\rangle|n\rangle$ and[1]

$$|\langle i|H'|f\rangle|^2 = \frac{\hbar\omega}{2\epsilon_0 V}\overline{\left|\langle g|\vec{d}\cdot\vec{u}|e\rangle\right|^2} \tag{12}$$

Integrating over all final photon directions[2] [1],

$$\rho_f = \frac{V}{2\pi^2}\frac{\omega_0^2}{\hbar c^3} \tag{13}$$

$$\Gamma = \frac{\omega^3}{2\pi\epsilon_0\hbar c^3}\overline{\left|\langle g|\vec{d}\cdot\vec{u}|e\rangle\right|^2} \tag{14}$$

Since spontaneous emission is caused by vacuum fluctuation, we average over all possible spatial orientations of $\hat{u}$ in Γ. We obtain one-third of the dipole matrix element in Ω for that in Γ if the optimal laser polarization is used to drive the transition.

Therefore, for optimally polarized light,

$$\sigma(\omega) \quad \propto \quad \frac{\Omega^2}{\Gamma} \tag{15}$$

$$= \quad \frac{3\lambda_0^2}{2\pi}\frac{\Gamma^2/4}{(\omega-\omega_0)^2 + \Gamma^2/4} \tag{16}$$

Note that on resonance, the cross section is independent of the transition strength.

$$\sigma(\omega_0) = \frac{3\lambda_0^2}{2\pi} \tag{17}$$

The result can be understood by noting that slower spontaneous decay in a narrow transition is compensated by a higher excitation probability on resonant.

Two-level atom

Next, we consider the case of a two-level atom with multiple m_J states. Suppose that both atom and laser are polarized. We can calculate the contribution of the total cross section from a specific ground state M_J.[3]

Let's first examine the Rabi frequency.[4]

$$\Omega^2 \quad = \quad \frac{n\omega_0}{\epsilon_0\hbar V}|\langle g, J, M_J|\vec{d}\cdot\hat{u}|e, J', M_J'\rangle|^2 \tag{18}$$

$$= \quad \frac{n\omega}{\epsilon_0\hbar V}|\langle g, J, M_J|du_q|e, J', M_J'\rangle|^2 \tag{19}$$

Applying the Wigner-Eckart theorem,

$$\Omega^2 = \frac{n\omega}{\epsilon_0\hbar V}\frac{|\langle g, J||d||e, J'\rangle|^2}{2J+1}|\langle J, M_J, 1, q|J', M_J'\rangle|^2 \tag{20}$$

The spontaneous emission rate is now an incoherent sum of decay rate through all possible channel. We can similarly apply the Wigner-Eckart theorem to each decay term.

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[1] We have used the energy density of the zero-point radiation field, $\epsilon_0 E^2 = \hbar\omega/2V$.

[2] We do not sum over polarizations because it has already been determined by the orientations of both $\vec{d}$ and $\hat{u}$.

[3] In absence of other processes, we will deplete the specific group M_J state through optically pumping, and a steady state will not be archieved. Therefore, at least part of the system, atom or laser field, must be unpolarized, and we will sum over different contributions to obtain the total cross section.

[4] We have chosen the polarizations such that $\vec{d}$ is along one of the basis vector $\hat{u}$.

$$\Gamma_{M'_J \to M_J} = \frac{\omega^3}{6\pi\epsilon_0\hbar c^3} \frac{|\langle g, J||d||e, J'\rangle|^2}{2J+1} \sum_q |\langle J, M_J, 1, q|J', M'_J\rangle|^2 \tag{21}$$

$$\Gamma_{tot} = \sum_{M_J} \Gamma_{M'_J \to M_J} \tag{22}$$

$$= \frac{\omega^3}{6\pi\epsilon_0\hbar c^3} \frac{|\langle g, J||d||e, J'\rangle|^2}{2J+1} \sum_{M_J} \sum_q |\langle J, M_J, 1, q|J', M'_J\rangle|^2 \tag{23}$$

$$= \frac{\omega^3}{6\pi\epsilon_0\hbar c^3} \frac{|\langle g, J||d||e, J'\rangle|^2}{2J+1} \tag{24}$$

Therefore,

$$\sigma(\omega) = \frac{3\lambda_0^2}{2\pi} \frac{\Gamma^2/4}{(\omega - \omega_0)^2 + \Gamma^2/4} |\langle J, M_J, 1, q|J', M'_J\rangle|^2 \tag{25}$$

In the dipole approximation, light does not change the electronic spin s. Let's check if our result enforces such selection rule, if L and s are well-defined.

$$|\alpha J M_J\rangle = \sum_i C_i |\alpha L M_L\rangle |S M_S\rangle \tag{26}$$

where C_i are the Clebsch-Gordan coefficients.

Inserting Eq. 26 into Eq. 20,

$$\Omega^2 \propto |\langle J, M_J, 1, q|J', M'_J\rangle|^2 \tag{27}$$

$$= \sum_{i,j} |C_i^* C_j \langle g, L, M_L|\langle S, M_S|\langle 1, q|e, L', M'_L\rangle|S', M'_S\rangle|^2 \tag{28}$$

$$= \sum_{i,j} |C_i^* C_j|^2 \cdot |\langle g, L, M_L|\langle 1, q|e, L', M'_L\rangle|^2 \cdot |\langle S M_s|S' M'_s\rangle|^2 \tag{29}$$

$$\propto \sum_{\substack{S, M_S, \\ S', M'_S}} \delta_{S,S'} \delta_{M_S, M'_S} \tag{30}$$

Since $\sigma \propto \Omega^2$, our result indeed respects the $\delta S = 0$ selection rule.

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[1] Dmitry Budker, Derek Kimball, and David DeMille. *Atomic physics: An exploration through problems and solutions.* Oxford University Press, USA, 2 edition, November 2008.

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