Reynolds Number in Buffer Gas Cell

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The following discussion is based on Sections 3.2.1 and 3.3.1 of Nick Hutzler's thesis. This document is intended to correct an order-unity inaccuracy and a dimensionality error in the Reynold's number vs. flow rate and aperture diameter relationship reported in the last paragraph of Section 3.3.1.

The Knudsen number of a beam of particles is defined as:

$$Kn = \lambda/d \approx (\sqrt{2} n_0 \sigma d)^{-1} \tag{1}$$

where λ is the mean free path of the buffer gas, d is a characteristic length scale, in our case the cell aperture diameter, n_0 is the buffer gas number density, and σ is the buffer gas collision cross section with itself.

The Knudsen number is related to the beam's Mach number, Ma, and the Reynolds number, Re, by the von Kármán relation

$$\frac{1}{2}(Kn)(Re) \approx (Ma). \tag{2}$$

Since the molecules exit the cell at approximately their thermal velocity, which is near the speed of sound for a monoatomic gas, we assume that $(Ma) \approx 1$ and we have

$$\frac{1}{2}(Kn)(Re) \approx 1. \tag{3}$$

Thus the Reynolds number is given by

$$Re \approx 2/(Kn) = 2\lambda/d. \tag{4}$$

The Reynold's number is therefore approximately twice the number of collisions occuring within the vicinity (one aperture diameter) of the aperture. Substituting in the relation for Kn from Eq. (1), we have

$$Re \approx 2\sqrt{2} n_0 \sigma d. \tag{5}$$

We know (Nick's thesis, Eq. 3.21) that under stagnation conditions, we can relate the number density, n_0, to the flow, f_0, via:

$$n_0 = \frac{4 f_0}{A \bar{v}_0} = \frac{16 f_0}{\pi d^2 \bar{v}_0}, \tag{6}$$

where A is the aperture area and $\bar{v}_0$ is the mean thermal speed of the buffer gas. If we substitute this relationship into Eq. (5), we obtain:

$$Re \approx \frac{32\sqrt{2}\sigma f_0}{\pi d \bar{v}_0}. \tag{7}$$

For a gas in thermal equilibrium (and obeying the Maxwell-Boltzmann distribution), $\bar{v}_0$ is given by (Nick's thesis, Eq. 3.5):

$$\bar{v}_0 = \sqrt{\frac{8 k_\mathrm{B} T}{\pi m}}, \tag{8}$$

where k_B is Boltzmann's constant, T is the temperature and m is the mass of the buffer gas atom. For a neon beam at 17 K or a helium beam at 4 K, one can verify that $\bar{v}_0 \approx 140$ m/s.

To obtain an easily convertible Re for a typical buffer gas beam, we will assume that $f_0 = 1$ SCCM $= 4.47 \times 10^{17}$ atoms/s and $d_a = 4.5$ mm $= 0.0045$ m. We will take the buffer gas collision cross section to be a typical value of $\sigma = 3 \times 10^{-15}$ cm^2 $= 3 \times 10^{-19}$ m^2. Plugging these values into Eq. (7), we have:

$$Re \approx \frac{16 \times (3 \times 10^{-19}\ \mathrm{m}^2) \times (4.47 \times 10^{17}\ \mathrm{atoms/s})}{(0.0045\ \mathrm{m}) \times (140\ \mathrm{m/s})} = 3.4. \tag{9}$$

Therefore, we have the following relation for a typical neon or helium buffer gas beam:

$$Re \approx 3.4 \times (f_0/1 \text{ SCCM}) \times (4.5 \text{ mm}/d). \tag{10}$$

Compared with the result in Section 3.3.1 of Nick's thesis (also reported in the 2010 ThO beam paper "A cryogenic beam of refractory, chemically reactive molecules with expansion cooling" in *PCCP*), this is a factor of 5 larger and has the correct dependence on d. This result invalidates the rule of thumb, also stated in Nick's thesis, that "for our beam source the relationship $Re \approx (f_0/1 \text{ SCCM})$ is a good approximation." Plugging in the correct numbers for a 17 K neon beam emitted from an aperture 5 mm in diameter, a better rule of thumb would be:

$$Re \approx 3 \times (f_0/1 \text{ SCCM}). \tag{11}$$