

Anapole Lens

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March 29, 2017

1 Setup

Here is a schematic of the beamline used in the anapole experiment: The molecules are considered to be emitted

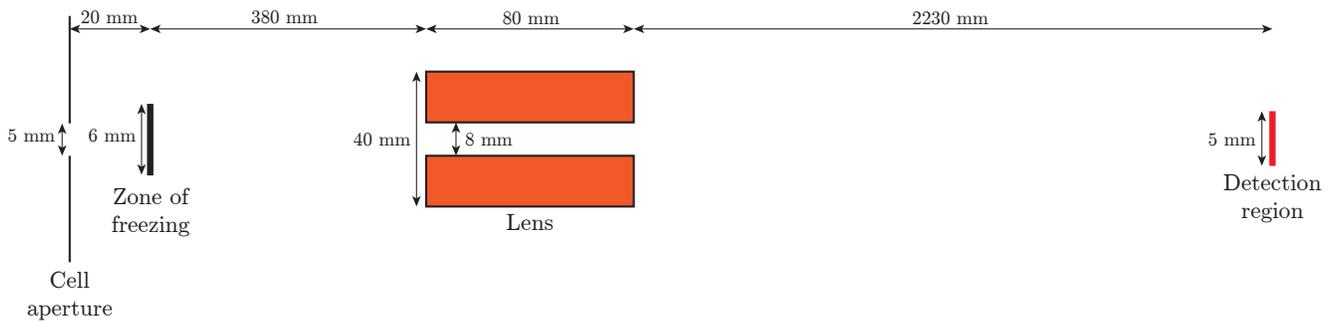


Figure 1: Beamline in the anapole experiment.

from an effective source at the zone of freezing, whose size is not well known. We assume there are no collimation apertures along the entire beamline. Molecules emitted then travel for some distance before reaching a hexapole lens. This lens is currently 8 cm in length, but we can consider changing that length. We can also consider varying the position of the lens. Molecules outside of the lens' inner radius are lost. After leaving the lens the molecules travel freely to a detection region, 5 mm in diameter. Molecules that arrive outside this diameter are lost.

For reference I include here a table with all the parameters of interest.

Quantity Description	Value	Unit	Variable Name in Code	Could vary?
Zone of freezing diameter	6	mm	zofd	✓
Lens inner radius	4	mm	borerad	✓
Lens outer radius	20	mm	outerrad	✓
Remanent field	1.3	T	Brem	✓
Number of Halbach segments	12		nseg	×
Gap between Halbach segments	0.1	mm	gap	✓
Distance from source to detection region	2.69	m	detz	✓
Diameter of detection region	5	mm	detd	✓
Magnetic sublevel	-1/2		M	×
Molecule mass	157	m_p	m	×
Molecule g-factor	2		g	×
Molecule beam forward velocity	616	m/s	fv	×
Forward velocity spread (Gaussian σ)	102	m/s	svz	×
Transverse velocity spread (Gaussian σ)	57	m/s	svx, svy	×
Distance from source to lens	0.38	m	s2l	✓
Lens length	8	cm	L	✓

Table 1: Parameters of interest in the anapole beamline/lens setup.

2 Magnetic Field

Details of the magnetic field produced from Halbach arrays is given in greater detail in my other writeups, e.g. for the ACME magnetic lens. Here I remind the reader that an analytic expression is given by *Nucl. Instr. Methods* **169**, 1–10 (1980). The resulting field for the anapole experiment geometry is shown in Fig. 2. A slice through this

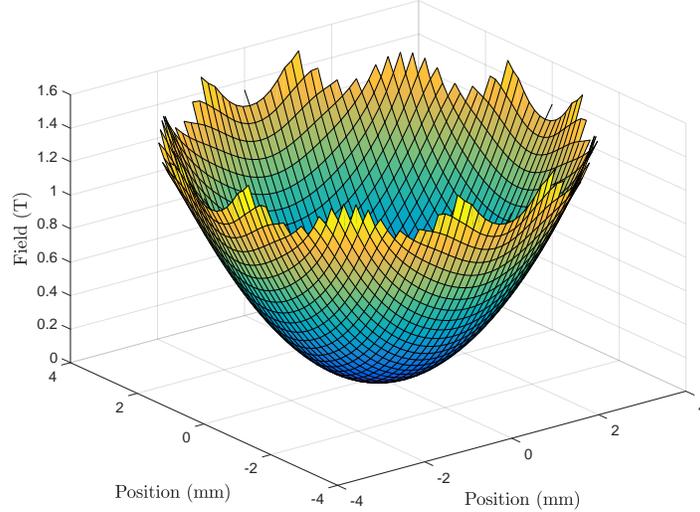


Figure 2: Magnetic field computed for the case of the anapole lens geometry, assuming infinite axial extent.

field, together with a purely quadratic fit, is shown in Fig. 3. The quadratic fit is of the form

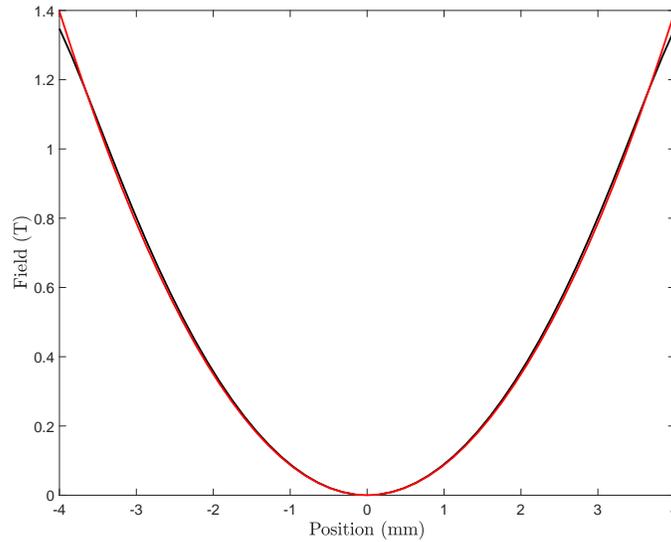


Figure 3: Slice through the magnetic field computed for the case of the anapole lens geometry (black), together with a quadratic fit (red).

$$B(r) \approx 8.7 \text{ T/cm}^2 \times r^2. \quad (1)$$

Note that the field levels off slightly towards the extrema. These calculations are based on a maximum field of 1.3 T reportedly measured by Emine. I assume that this is equivalent to the remanent field of the magnets (and is

in line with the spec'd value for N42 NIB), but if it is the maximum value in the Halbach configuration then these calculations may be overestimating the field.

3 Lens Focal Length

Following the treatment in *J. Phys. Chem.* **95**, 81298136 (1991) we can now calculate the focal length of the lens, and hence where we expect it to focus given the current configuration.

The potential is given by $V \approx \mu_B B = 8.1 \times 10^{-19} \text{ J/m}^2 \times r^2$. The force is given by $F = -\vec{\nabla}V = -kr$ from which we find that $k \approx 1.6 \times 10^{-18} \text{ N/m}$. The oscillation frequency is then given by $\omega = \sqrt{k/m} \approx 2.5 \text{ kHz}$.

The 'lens constant' is given by

$$p = \omega/v_{\parallel} = 2.5 \text{ kHz} \times 616 \text{ m/s} \approx 4.0 \text{ m}^{-1} \quad (2)$$

where v_{\parallel} is the forward velocity. The focal length of the lens is then given by

$$f = [p \sin(pL)]^{-1} \approx [4 \sin(4 \times 0.08)]^{-1} = 0.8 \text{ m}. \quad (3)$$

We can now estimate where the molecules will be focussed. The distance from source to lens entrance is a , the distance from lens exit to focal point is b . The principal planes of the lens are a distance d from the entrance and exit. One can show that

$$b' = \frac{a'f}{a' - f} = \frac{(a + d)f}{a + d - f} \quad (4)$$

where

$$d = \frac{1 - p \cos(pL)}{p \sin(pL)} = f[1 - p \cos(pL)]. \quad (5)$$

Then, $b = b' - d \approx -0.9 \text{ m}$. This is quite alarming; it tells us that the image is a virtual one, i.e. the lens is not focussing the molecules. It actually makes sense intuitively: if our lens focal length is 0.8 m and it is situated 0.38 m from the source, then it will not be able to focus as the lens is not strong enough.

How can we fix this? The easiest thing to do is to move the lens. We would like the current beamline length to be equal to $a + L + b$ where, to remind the reader, a is the source to lens distance, L is the lens length and b is the lens to focus distance. The figure below shows how the overall length varies while varying the distance from source to lens, keeping the lens length constant at 8 cm. We see that we have to move the lens a long way downstream in order to get the beam focussing near the detection region — a source to lens distance of around 1.5 m looks favourable.

4 Trajectory Simulations

Again, details of these simulations are available in my other writeups — contact me if you'd like them.

The result of Section 3 is echoed in my trajectory simulations. The plot below shows trajectories using the analytic form of the field for a Halbach array, and a point source with single forward velocity. We see that no focussing occurs. If I now increase the length of the lens I can make it start focussing. This is shown in the figure below. We see as expected that the molecules are focussed to a point (the couple of trajectories which seem to 'miss' are those that explore the edge of the lens bore, where the potential begins to deviate from quadratic). If I now perform the 'full' simulation I can corroborate the result seen in Fig. 4. I calculate the gain achieved while varying the source to lens distance. The results are shown in Fig. 7. We see that there is a peak in the gain at around 1.5 m, the same optimum distance as in Fig. 4.

5 Improving Performance

In practice, it will probably not be feasible to move the lens downstream in the way shown in the previous section. Better choices for improving the performance would be to either move the beam source further upstream, or to lengthen the lens in the upstream dimension. I now consider these options.

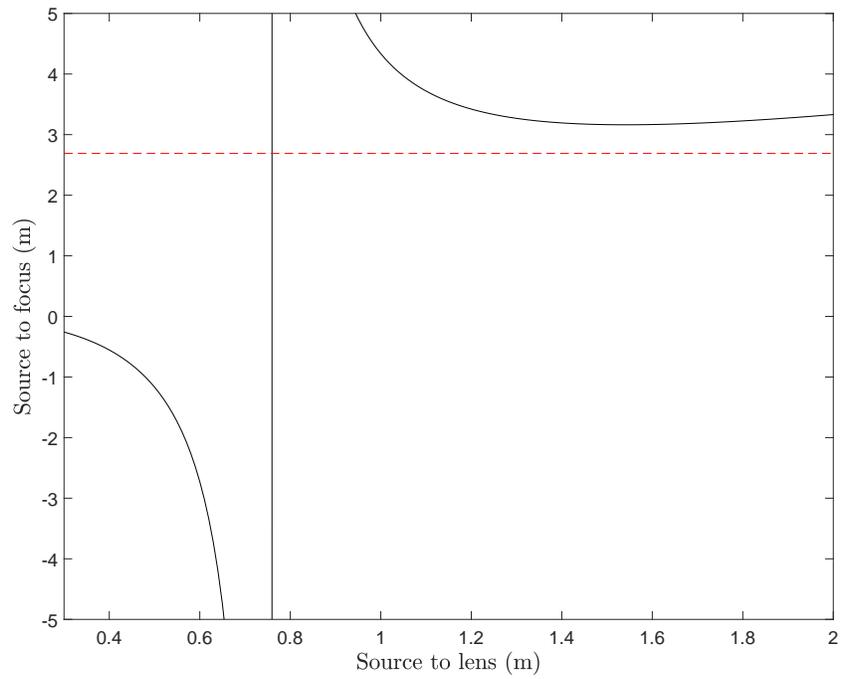


Figure 4: Analytic calculation of the distance from beam source to beam focal point as a function of the distance from source to lens, keeping the lens length constant at 8 cm. The red dashed line shows the current distance.

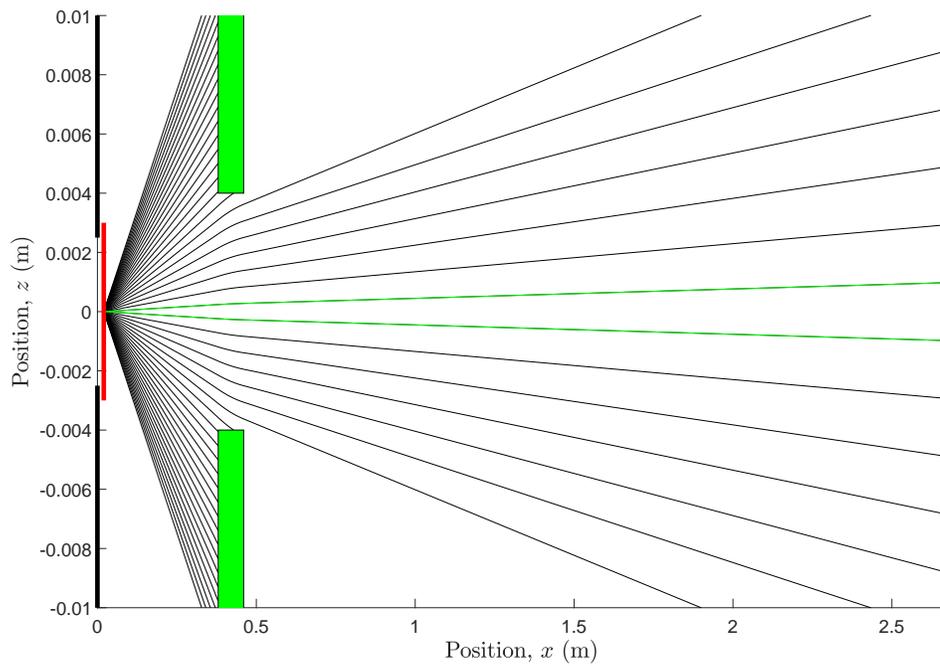


Figure 5: Molecule trajectories assuming a point source and single forward velocity.

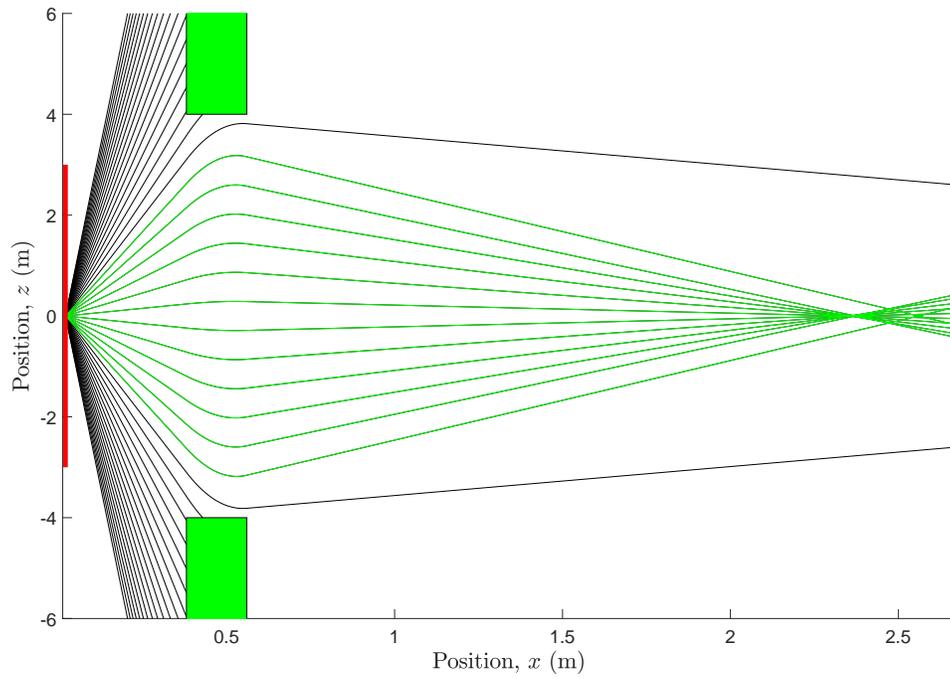


Figure 6: Molecule trajectories assuming a point source and single forward velocity. The lens length has been increased to show focussing.

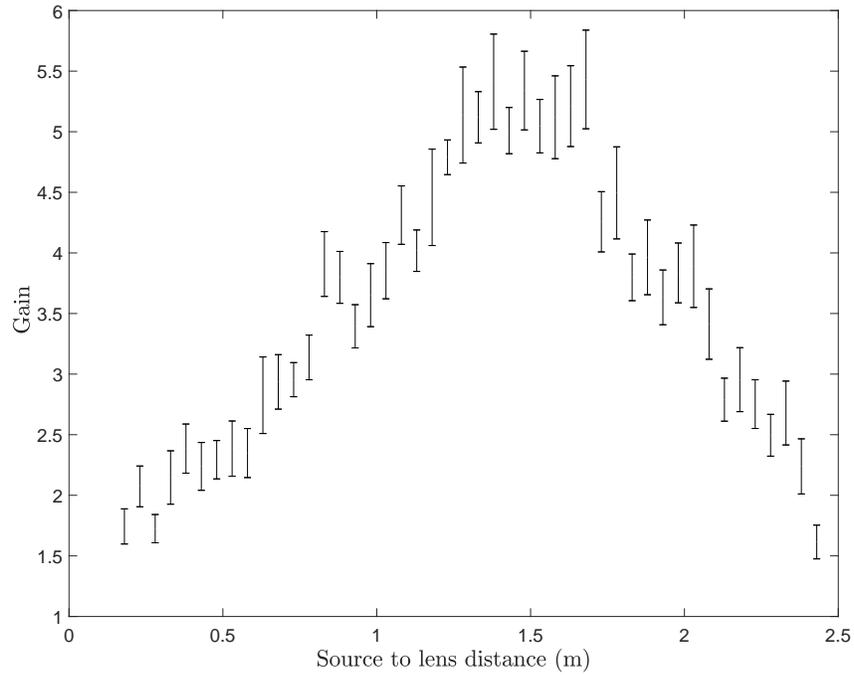


Figure 7: Gain achieved with the lens as a function of the distance between the source and the lens.

5.1 Increasing distance from source to lens

Here I consider the effect of moving the molecule source away from the lens, i.e. the lens length stays fixed (8 cm), the distance from the end of the lens to the detection region stays fixed (2.23 m) and the source moves further upstream from the lens start. I also consider moving the source a little closer to the lens start for completeness.

Fig. 8 shows the dependence of the gain on the distance from the beam source to the lens. The current distance is 0.38 m. Error bars are standard error on the mean for multiple simulation runs. We see that there is in general

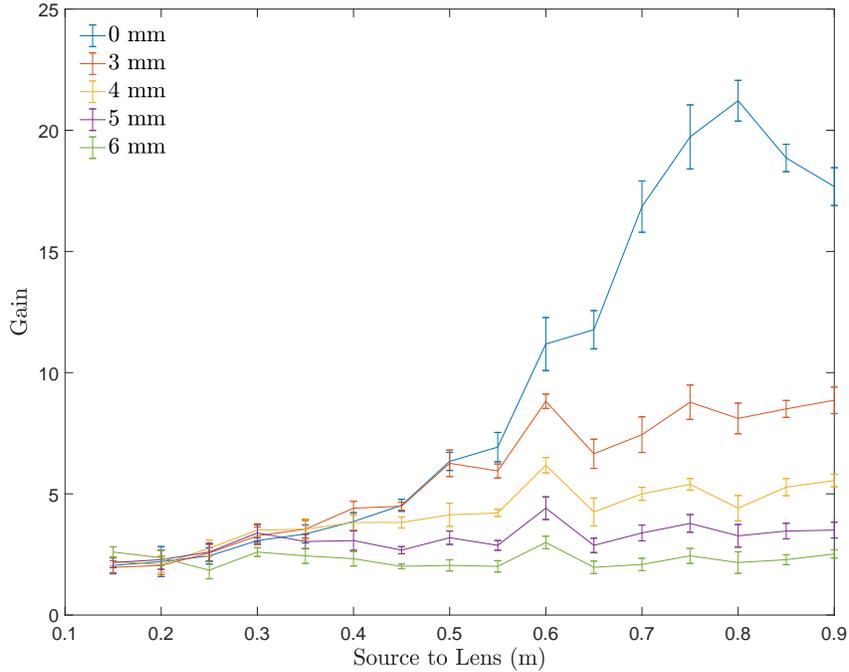


Figure 8: Dependence of the molecule number gain on the distance between molecule source and lens start, for different effective source sizes. See main text for description.

an increase in the gain with the distance from source to lens, with peak performance observed for a distance of around 0.8 m. This is as expected as we calculated the lens focal length to be around 0.8 m. We also see that there is a significant reduction in performance as the effective source size is increased. For a 3 mm diameter source the maximum gain is around a factor of 6, but for a 6 mm diameter source the gain is only a factor of 2, with no observed dependence on the distance between source and lens over the range examined.

5.2 Increasing lens length

Next we consider lengthening the lens itself. In doing so, we keep the distance from the end of the lens to the detection region constant, and we keep the position of the source the same, so the distance from the source to the lens is reduced by the increase in the lens length compared to the current value of 8 cm. The resulting data are shown in Fig. 9. We see that there is an increase in gain as the lens length is increased, however it is again

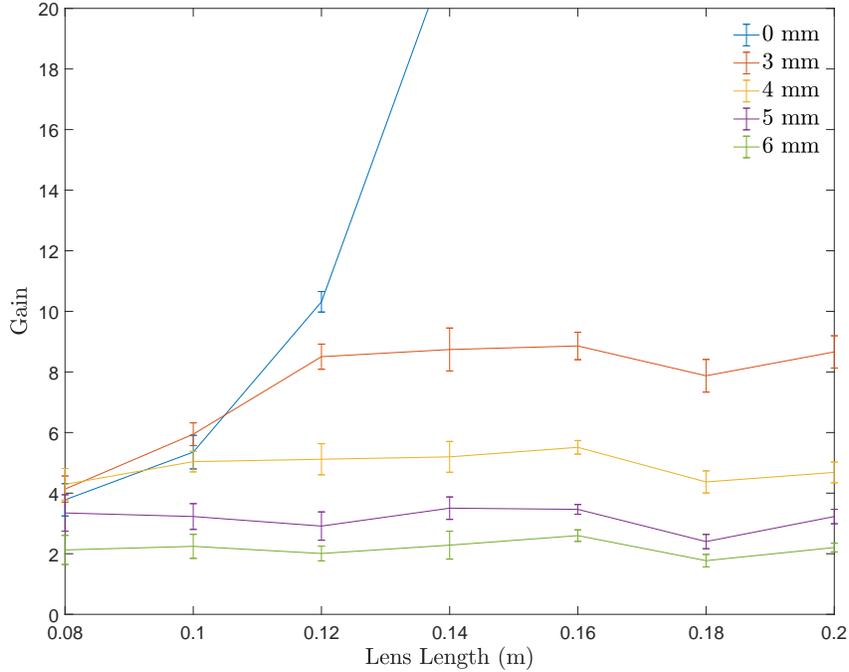


Figure 9: Dependence of the molecule number gain on the lens length, for different effective source sizes. See main text for description.

strongly dependent on the source size. For a point source, a huge gain can be obtained (maximally 100 for the range considered here), but a 3 mm diameter source yields a gain of only around 9. For a 6 mm source, no improvement is seen by changing the lens length.

5.3 Source to lens distance and lens length

We can now consider changing both of the parameters from the previous two sections in order to optimise performance, i.e. varying both the lens length and distance from source to lens simultaneously. This was done first of all assuming a source size of 6 mm, as this is perhaps our best guess of the source size based on the observed gain from the current lens configuration. The results are shown in Fig. 10. We see that there is rather little change in the

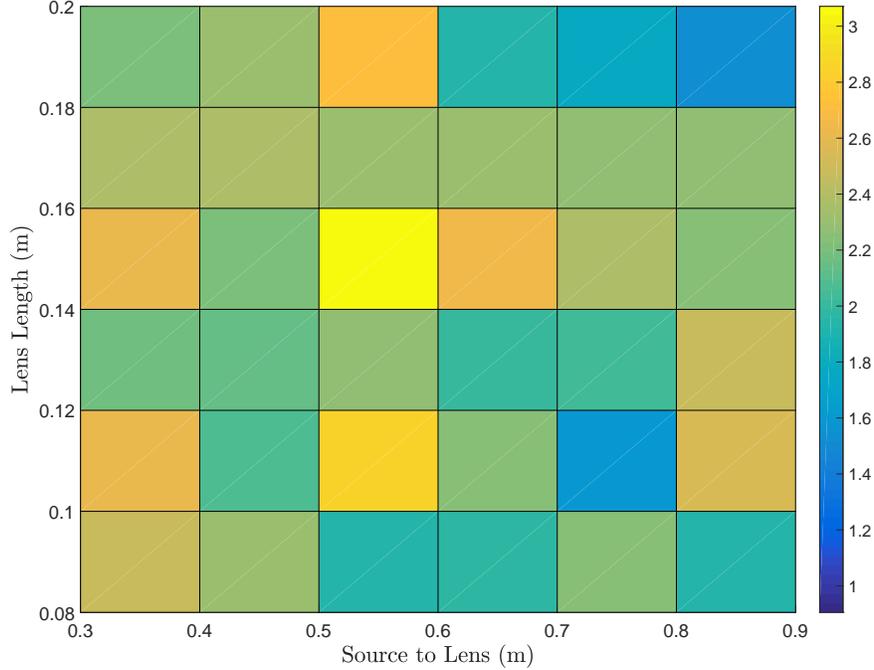


Figure 10: Dependence of the molecule number gain on the lens length and distance from molecule source to lens, assuming a source size (diameter) of 6 mm.

gain over the whole parameter space explored; the gain seems to be approximately constant within the associated noise.

We now repeat this analysis for a source size of 3 mm. The results are shown in Fig. 11. Here we see a much more significant increase in the gain, with a maximum value of around 9. We also see that there is a clear relationship between the required source to lens distance and lens length; as the lens length increases, its effective focal length decreases and the distance from source to lens must decrease in order for the molecule beam to remain approximately collimated.

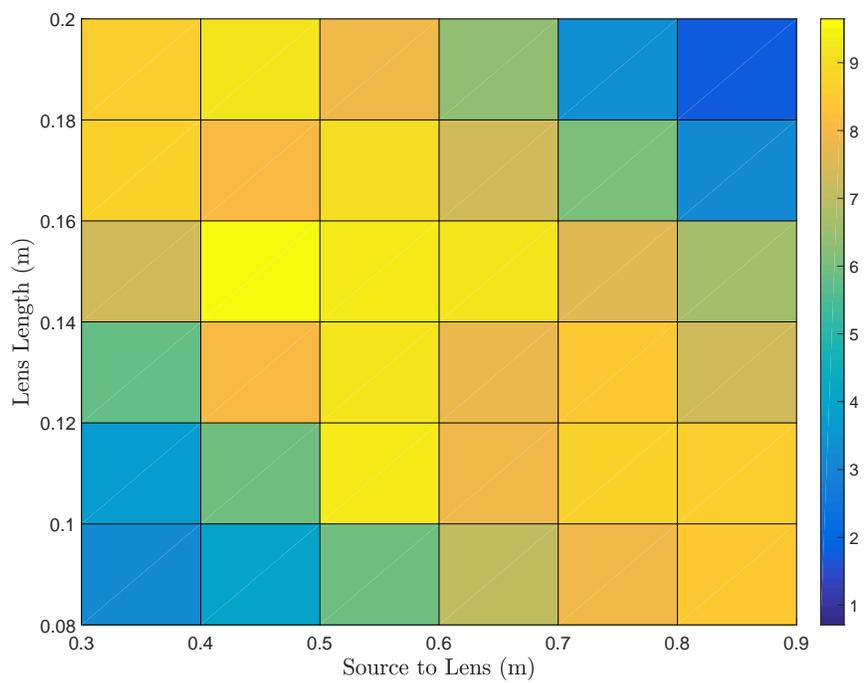


Figure 11: Dependence of the molecule number gain on the lens length and distance from molecule source to lens, assuming a source size (diameter) of 3 mm.

5.4 Lens diameter

We now consider the option of increasing the inner diameter of the lens. Since the performance seems to be severely limited by the finite source size, increasing the diameter of the lens should bring us closer to the approximation of a point source. However, this comes at a cost: the curvature of the magnetic field decreases quadratically with the lens diameter. To compensate for this and keep the lens focal length constant, the lens length must be increased quadratically.

To illustrate this we consider the following: In Fig. 9 we see that for a 3 mm source size and the current source to lens distance (38 cm), a lens length of around 15 cm gives the optimum gain. This uses the default inner diameter of 8 mm. If we now increase this diameter to 12 mm we would expect the optimum lens length to increase to around $15 \text{ cm} \times (12 \text{ mm}/8 \text{ mm})^2 \approx 34 \text{ cm}$. Simulating the achievable gain for this larger diameter, as a function of lens length gives the data shown in Fig. 12. We see that in this configuration the gain is maximised at a lens length of

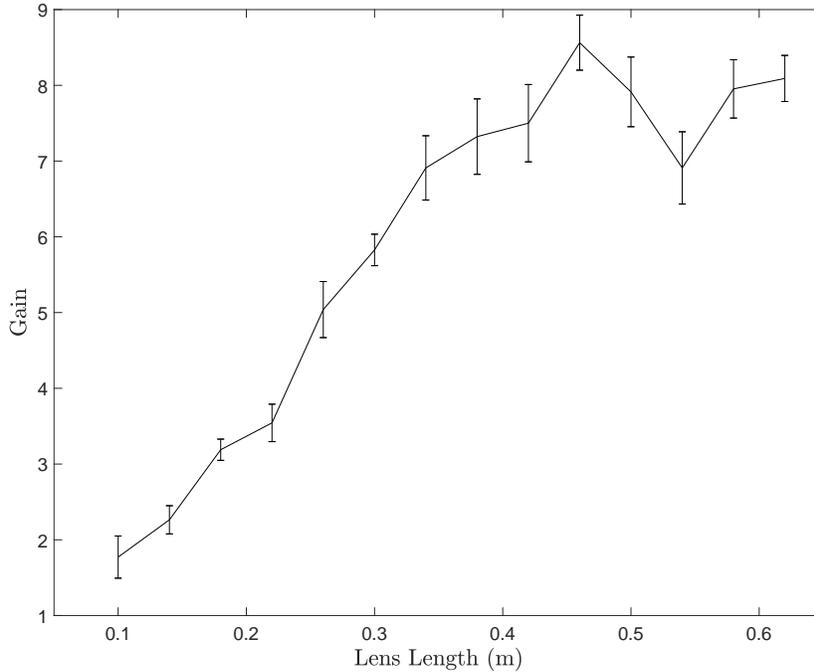


Figure 12: Dependence of the molecule number gain on the lens length, assuming a source size (diameter) of 3 mm and a lens inner diameter of 12 mm.

around 40 cm, approximately in line with our rough scaling argument; a considerable increase in length is required to achieve around the same gain. For these simulations an outer lens diameter of 12 in is used. Note that the outer diameter has a very small effect on performance over this range.

If we repeat this calculation with a source size of 6 mm we find that we achieve a greater gain. Fig. 13 shows that increasing the lens diameter allows a gain of around 5 after optimising the lens length, whereas with an 8 mm bore (Fig. 9) the gain is limited to 2.

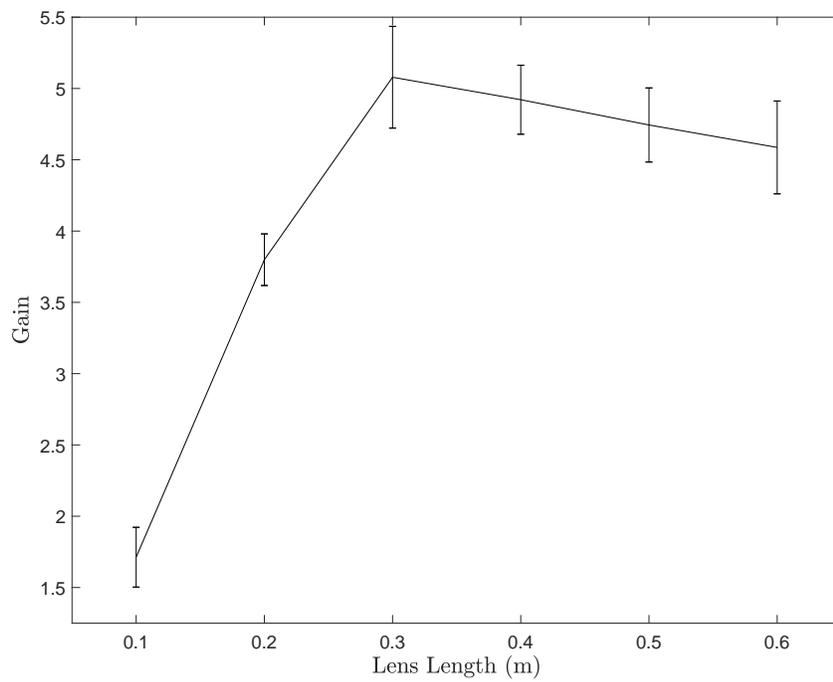


Figure 13: Dependence of the molecule number gain on the lens length, assuming a source size (diameter) of 6 mm and a lens inner diameter of 12 mm.

5.5 Forward velocity

The molecule source currently used in the anapole experiment has a rather high forward velocity, around 600 m/s. This is perhaps undesirable. For a given transverse velocity it requires a larger longitudinal distance for molecules to reach a particular radial position. In the previous section we noted that increasing the lens diameter would make the molecule source better approximated as a point source, and observed improved performance by doing so. However, this required a significantly longer lens. By using a slower beam source we may be able to reduce that length.

To examine this possibility I consider changing the molecule beam properties such that they match those in ACME. I.e. a mean forward velocity of 200 m/s, a $1\text{-}\sigma$ forward velocity spread of 17 m/s and a transverse velocity $1\text{-}\sigma$ of 57 m/s. Changing the forward velocity alters the analytically calculated lens focal length for the current geometry from 79 cm to 10 cm — a huge change! We could then reduce the lens length to 1.75 cm and get a focal length equal to the source-to-lens distance of 38 cm.

However, the lower forward velocity has unwanted effect as well: it reduces the effective geometric acceptance of the lens; the transverse velocity cut off is now lower. So, let's calculate what transverse velocities we can capture based on the potential depth and then work out how big a bore we should have so that they are also geometrically captured. For a 1.3 T remanent field, the maximum field is around 1.55 T. This corresponds to a potential depth of

$$V_{\max} = Mg\mu_B B_{\max} = 0.5 \times 2 \times \mu_B \times 1.55 \text{ T} \approx 1 \text{ K} \times k_B. \quad (6)$$

We can then convert this to a velocity:

$$\frac{1}{2}mv^2 = 1 \text{ K} \times k_B \quad (7)$$

$$\Rightarrow v = \sqrt{2 \text{ K} \times k_B/m} \approx 10 \text{ m/s}. \quad (8)$$

If the lens is 38 cm from the source it will take 1.9 ms to reach the lens. In 1.9 ms it will travel 1.9 cm in the transverse direction. If our source size is 6 mm then the total transverse displacement of the fastest molecules within our capture depth is 2.2 cm. Thus we want our lens to have an inner diameter of around 4.4 cm in order to geometrically capture as many molecules as possible.

As an aside, if we do this calculation for a forward velocity of 616 m/s, we want a lens diameter of 1.2 cm — bigger than the current diameter of 0.8 cm.

If I assume a forward velocity of 200 m/s and a lens inner diameter of 4.5 cm, I can examine the lens focal length as a function of the lens length, shown in Fig. 14. In order to make the focal length as close to the distance between source and lens as possible the lens length has to be around 65 cm. This is a rather unwieldy length. Note that the minimum focal length of the lens is also the longitudinal distance travelled by the largest capturable velocity class as it travels a transverse distance equal to the lens inner radius. One could imagine moving the lens further from the source to make the source-to-lens distance match the focal length better, but then the lens would capture fewer molecules.

To reiterate, we want:

- Source-to-lens distance equal to lens focal length
- Source-to-lens distance less than distance travelled by 'capture velocity' molecules

To fix this we can consider reducing the lens inner diameter. Repeating the calculation shown in Fig. 14 for a range of different lens inner diameters produces the results in Fig. 15. We see, as expected, that the lens focal length decreases significantly as the lens inner diameter is decreased. It is then up to us to decide where on this set of curves we wish to sit. We could pick a lens length, say 20 cm, in which case we have a range of possible lens diameters and distances from the source, or we could pick a distance from the source, in which case we have a range of possible lens inner diameters and lengths. By way of a reasonable example I will pick a lens length of 16 cm and an inner diameter of 1 in. For this pair of parameters the lens focal length is calculated to be 40 cm — very close to the current distance.

Let's now try putting these parameters into the trajectory simulations. As a quick check, we see in Fig. 16 that molecules emitted from a point source are pretty well collimated. I now repeat the simulations with a realistic beam source (diameter of 6 mm and Gaussian distribution of transverse velocities). Note that because the lens inner diameter is now bigger than the detection region size, the optimum configuration is probably weak focussing of the molecule beam. As such, I varied the distance between the source and lens to try and find an optimum. The resulting data are shown in Fig. 17. We see that there is not a significant improvement in the gain compared to

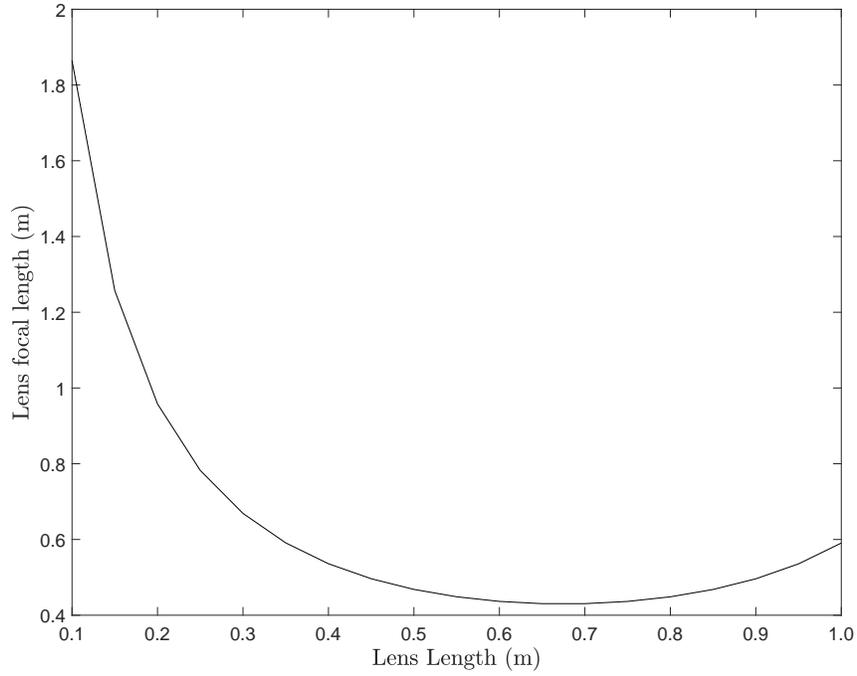


Figure 14: Dependence of the lens focal length (analytically calculated) on the lens length, assuming a 200 m/s forward velocity and a 45 mm lens inner diameter.

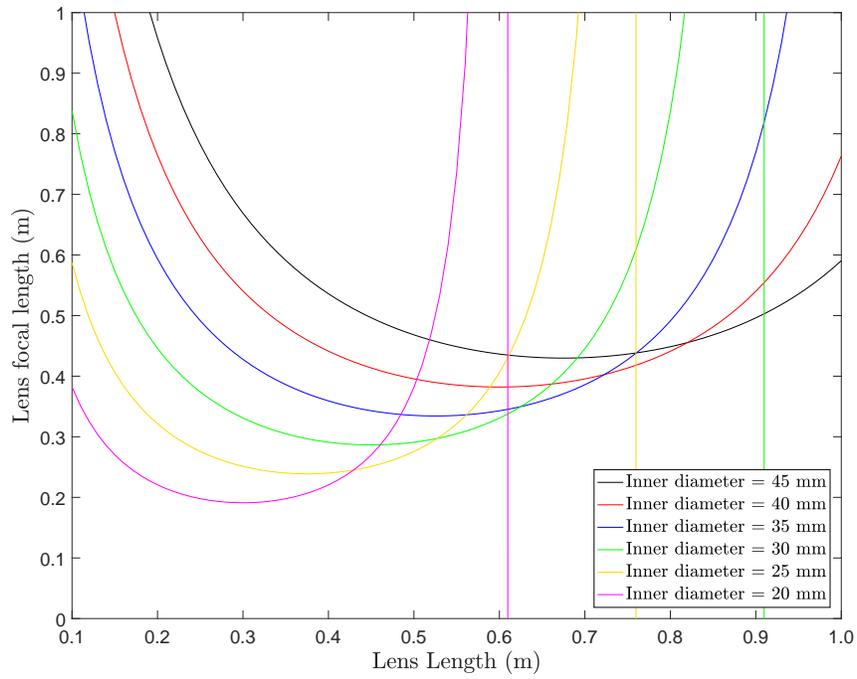


Figure 15: Dependence of the lens focal length (analytically calculated) on the lens length and lens inner diameter, assuming a 200 m/s forward velocity.

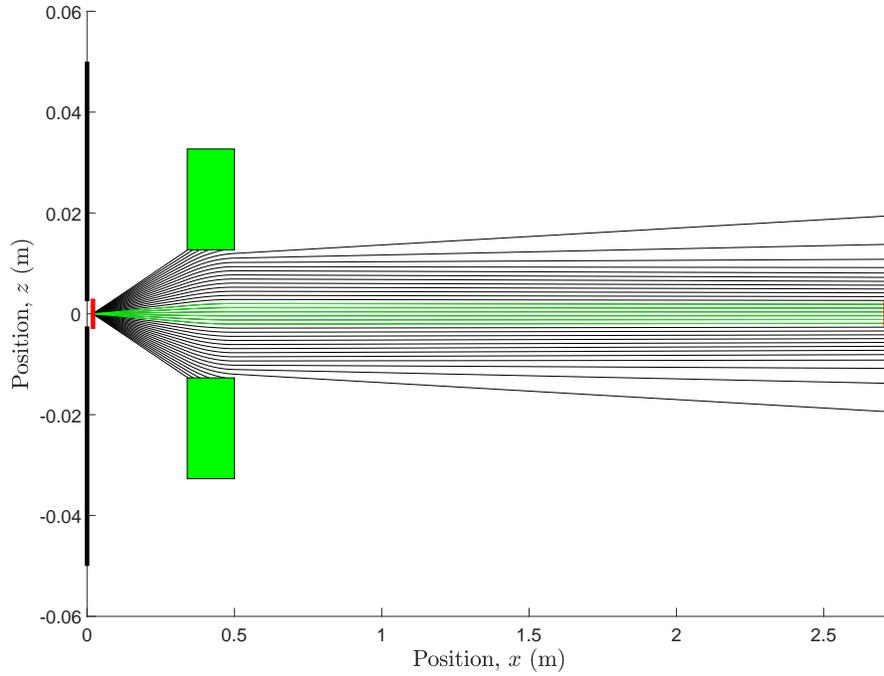


Figure 16: Example trajectories for a lens length of 16 cm, lens inner diameter of 0.5 in, source to lens distance of 40 cm and a point source of molecules. We see good collimation indicating that the lens is appropriately set up.

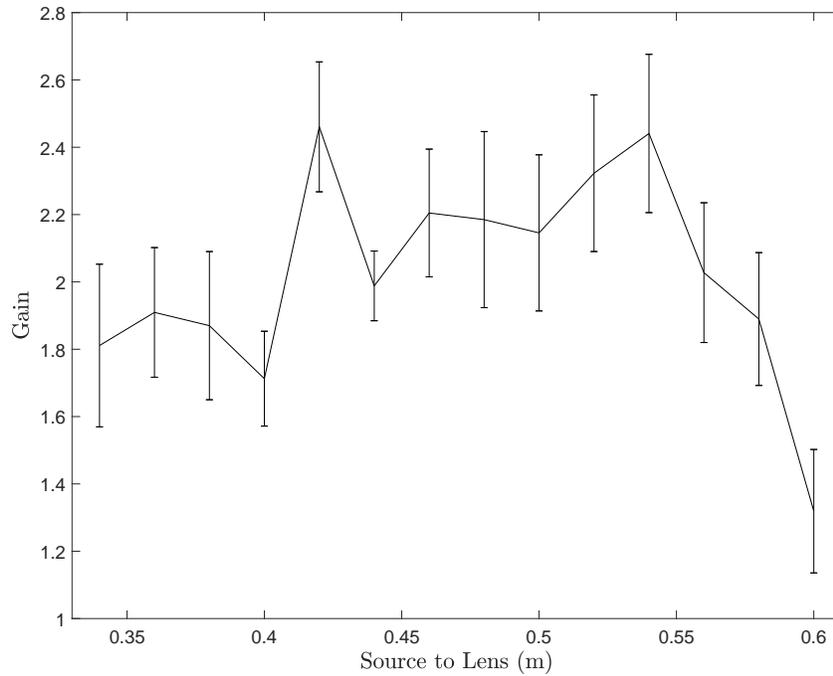


Figure 17: Gain as a function of the distance between the beam source and lens. The lens length is 16 cm, the inner diameter is 1 in and the molecule beam is assumed to come from a buffer gas source with a source size of 6 mm.

our tests with the ‘fast’ beam — perhaps a slight improvement. Repeating this analysis with a 3 mm source size gives Fig. 18. We see that the gain we can achieve is in fact a little less than with the faster beam and smaller lens diameter. It is also worth noting that the source size from a buffer gas source is unlikely to be significantly smaller

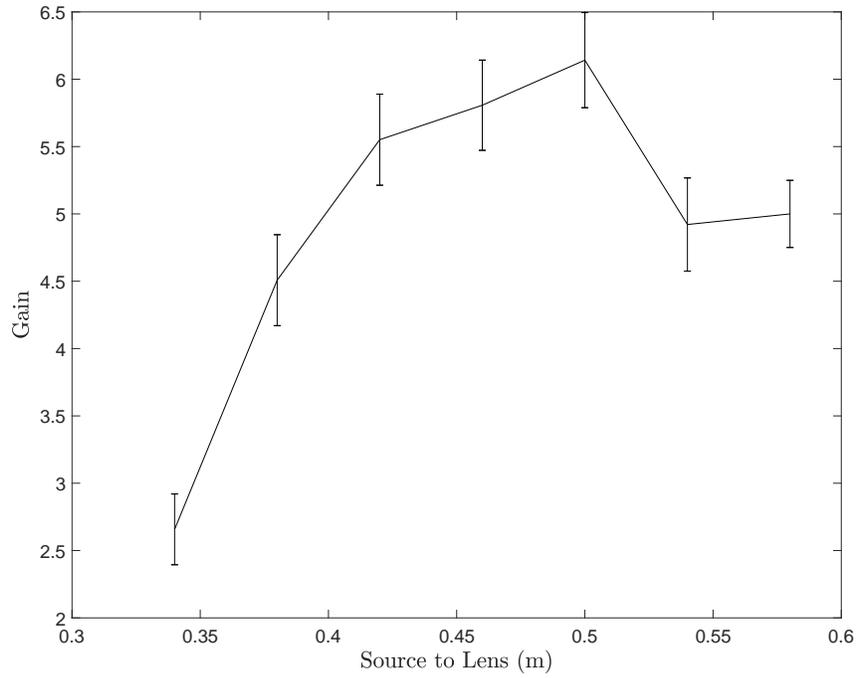


Figure 18: Gain as a function of the distance between the beam source and lens. The lens length is 16 cm, the inner diameter is 1 in and the molecule beam is assumed to come from a buffer gas source with a source size of 3 mm.

than 6 mm.