Systematic Error Determination

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We will include a systematic effect from varying a parameter X in our systematic uncertainty if one of the following criteria are met:

A) We have directly seen the EDM change as a function of X.

B) We have seen unexplained behavior in one of the following non-EDM channels: $\omega^{\mathcal{N}}, \omega^{\mathcal{E}}, \omega^{\mathcal{E}\mathcal{B}}, \omega^{\mathcal{N}\mathcal{E}\mathcal{B}}, \eta$.

C) We have a parameter X that is physically analogous to a parameter in a previous experiment X', and the previous experiment saw correlations between d_e and X' that were not fully understood, or whose implications in our experiment are not understood.

Table 1 contains a list of d_e systematic errors for parameters that met any of these criteria.

A: Direct EDM Change. If we have measured a non-zero slope of dd_e/dX, then we calculate the corresponding systematic correction and uncertainty in the following way:

1. Compute the mean slope $S = dd_e/dX$ and slope uncertainty δS.
2. Find the mean $\bar{X}$ and uncertainty δX, either from an auxiliary measurement or from a parity channel monitored in situ.
3. The systematic uncertainty is then $\delta d_e = [\bar{X}^2(\delta S)^2 + (\delta X)^2 S^2]^{1/2}$, and the EDM shift is $S\bar{X}$.

There are three systematics in this category: $\mathcal{E}^{\mathrm{nr}}, P^{\mathcal{N}\mathcal{E}}$, and $\mathcal{B}^{\mathcal{E}}$. We note that our method for calculating a $\mathcal{B}^{\mathcal{E}}$ systematic error also bounds $v \times \mathcal{E}$ and geometic phase type systematics, which were observed and well understood in past EDM experiments.

B: Unexplained Parity Sum Behavior. There are two types of unexplained behavior in non-EDM channels that we include in this category:

Drifting parameter. If any of the superblock-even $\omega^{\mathcal{N}}, \omega^{\mathcal{E}}, \omega^{\mathcal{E}\mathcal{B}}, \omega^{\mathcal{N}\mathcal{E}\mathcal{B}}, \eta$ channels show a statistically significant "drift" (χ^2 or autocorrelation $> 4\sigma$ from expected value), then we include a systematic uncertainty for that channel. Let X be the value of the drifting channel.

1. Assume a linear relationship between X and d_e and extract the slope $S = dd_e/dX$ and slope uncertainty δS from a line fit. We only deal with superblock-even parity sums.
2. Take the final mean $\bar{X}$ and uncertainty δX over all final data runs, and any runs where the effect was purposely exaggerated.
3. The systematic uncertainty is then $\delta d_e = [\bar{X}^2(\delta S)^2 + (\delta X)^2 S^2]^{1/2}$. We do not perform a subtraction.

There is one systematic which falls under this category, $\omega^{\mathcal{N}}$. We observed that under certain conditions the AOMs used for fast polarization switching generated a k^N of 5 μrad. We tuned k^N by $+/-$ 100 μrad with mirrors mounted to piezoelectric adjusters and observe a >4 sigma dependence of ω^N on k^N that fluctuated significantly. To determine a d_e systematic error for effects coupling to k^N causing significant fluctuations in ω^N, we looked for correlations between d_e and the fluctuating ω^N channel. We obtained an upper limit of $24 \times 10^{-30} e \cdot cm/(rad/sec)$ for this correlation. Plugging in an average ω^N value of 0.035 rad/sec measured during our final data sets, we obtained a systematic error of 8.0×10^{-30} e·cm.

Linear dependence. If any of the superblock-even $\omega^{\mathcal{N}}, \omega^{\mathcal{E}}, \omega^{\mathcal{E}\mathcal{B}}, \omega^{\mathcal{N}\mathcal{E}\mathcal{B}}, \eta$ channels show a statistically significant linear dependence vs. a parameter X, then we include a systematic uncertainty for the parameter X.

1. Assume a linear relationship between X and d_e and extract the slope $S = dd_e/dX$ and slope uncertainty δS. We only deal with superblock parity sums.

2. Find the mean $\bar{X}$ and uncertainty δX, typically from an auxiliary measurement.

3. The systematic uncertainty is then $\delta d_e = [\bar{X}^2(\delta S)^2 + (\delta X)^2 S^2]^{1/2}$. Do not perform a subtraction.

If we set our cutoff at 4σ statistical significance, then no systematics will enter into this category.

C: Previously Observed Effects. Often, a parameter X' in a previous experiment has a physically analogous parameter X in our experiment. If a previous atom/molecule EDM experiment (YbF, PbO, Tl, Hg) observed a systematic dependence of the EDM on X' that was not explained, or if we do not fully understand how an analogous effect dependent on X could manifest itself in our experiment, then we will include that systematic effect in our uncertainty. Again, a linear model is assumed and the EDM uncertainty is calculated as discussed in (B). There are three systematics in this category.

1. Stray magnetic fields. The PbO experiment saw large dependence of the EDM on various stray magnetic fields, including transverse fields and gradients. Therefore, we will include (B-even) transverse fields and gradients: $B^{\mathrm{nr}}, B_x, B_y$, and all B-even gradients.

2. Detuning effects. The YbF experiment observed a number of detuning-related systematic effects (for example, detuning coupling to spatially varying electric field), and we do not have a complete model for how these effects might manifest themselves in our system. Therefore, we will include detuning-related systematics. These include $\Delta^{(0)}_{\mathrm{prep}}, \Delta^{(0)}_{\mathrm{read}}, \Delta^{\mathcal{N}}$, and $\Delta^{(0)}\Delta^{\mathcal{N}}$.

3. Non-zero field plate voltage. The YbF experiment observed an unexplained variation of their EDM on the mean (“floating”) voltage applied to their field plates. Therefore, we include a systematic error for V_{offset}.

References

Class	Parameter X	Shift (10^{-30} e cm)	Uncertainty (10^{-30} e cm)
A	$P^{\mathcal{N}\mathcal{E}}/P^{(0)}$	.3	12.7
	$\mathcal{E}^{\rm nr}$	11.3	5.1
	$\omega^{\mathcal{E}}$	$-.08$	.09
B	$\omega^{\mathcal{N}}$		6.0
C	$\mathcal{B}_x^{\rm nr}$		.1
	$\mathcal{B}_y^{\rm nr}$		2.3
	$\mathcal{B}_z^{\rm nr}$		3.2
	$\frac{\partial \mathcal{B}_x^{\rm nr}}{\partial x}$		2.0
	$\frac{\partial \mathcal{B}_y^{\rm nr}}{\partial x}$		1.8
	$\frac{\partial \mathcal{B}_y^{\rm nr}}{\partial y}$		2.5
	$\frac{\partial \mathcal{B}_y^{\rm nr}}{\partial z}$		3.0
	$\frac{\partial \mathcal{B}_z^{\rm nr}}{\partial x}$		2.2
	$\frac{\partial \mathcal{B}_z^{\rm nr}}{\partial z}$		3.1
	(total $\mathcal{B}_i^{\rm nr}$, $\frac{\partial \mathcal{B}_i^{\rm nr}}{\partial x_j}$)		(7.2)
	$\Delta^{(0)}_{\rm prep}$		7.7
	$\Delta^{(0)}_{\rm read}$		4.4
	$\Delta^{\mathcal{N}}$		6.8
	$\Delta^{(0)}\Delta^{\mathcal{N}}$		2.3
	(total Δ_i^p)		(11.4)
	$V_{\rm offset}$		1.0
Total A		1.5*	13.6
Total B			6.0
Total C			13.5
Total Systematic		1.5	20.1
Statistical			30.5
Stat. and Syst.			36.5

Table 1: Table of Systematic Errors grouped by class. *For the class A systematics, we perform the systematic subtraction on a state by state basis for the $\hat{k} \cdot \hat{z}$ and $\mathcal{B}$ magnitude states. Hence the effective shift derived for each individual systematic is different from the total class A effective shift.